MAT 2384 C Assignment 5

Winter 2019, Dr. Schmah

due Wednesday, April 3rd, 4:00pm, in class

Students are encouraged to discuss homework problems with others. However, once you understand how to solve a problem, you must write out your own solution, without copying others' work.

[10pts] 1. Find the general solution to the following nonhomogeneous system:

$$\mathbf{y}' = \begin{bmatrix} 2 & -4 \\ 4 & -6 \end{bmatrix} \mathbf{y} + \begin{bmatrix} -2x^2 + 14x + 14 \\ -4x^2 + 18x + 23 \end{bmatrix}$$

[15pts] 2. Find the Laplace transforms of:

(a) $f(t) = \begin{cases} e^t, & 0 < t \leq \pi/2, \\ 0, & t \geq \pi/2. \end{cases}$

(b) $f(t) = te^{-4t}u(t-3)$.

(c) $f(t) = \sin(\omega t + \theta)$.

[10pts] 3. Calculate:

(a) $e^t * e^{-t}$

(b) $\mathcal{L}^{-1}\left\{6\left(1 - e^{-\pi s}\right) / \left(s^2 + 9\right)\right\}$

[10pts] 4. Solve the initial value problem $y'' + y = \delta(t-\pi) - \delta(t-2\pi), \quad y(0) = 0, y'(0) = 1.$

[15pts] 5. Consider the initial value problem $y' = x + 2\cos y, \quad y(0) = 0.$

(a) Use the Improved Euler's method, with $h = 0.1$, to approximate the solution of the IVP on the interval $0 \leq x \leq 0.4$. Calculate the absolute error of the estimate of $y(0.4)$.

(b) Use the fourth order Runge-Kutta method, with $h = 0.2$, to approximate the solution of the same IVP on the same interval $0 \leq x \leq 0.4$. Calculate the absolute error of the estimate of $y(0.4)$.

Round your results to 6 decimal places at each step.