MAT 2384 C Assignment 4

Winter 2019, Dr. Schmah

due Monday, March 18th, 5:30pm, in class

Students are encouraged to discuss homework problems with others. However, once you understand how to solve a problem, you must write out your own solution, without copying others' work.

[5pts] 1. Solve the initial value problem:

$$\mathbf{y}' = \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} \mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

[7pts] 2. Find the general solution of the system:

$$y_1' = -\frac{1}{2} y_1 + y_2$$
$$y_2' = -y_1 - \frac{1}{2} y_2$$

[10pts] 3. Two tanks hold 100 litres of liquid each. The first tank starts with 25 kg of dissolved salt, while the second starts with pure water. Liquid flows from tank 1 into tank 2 at 4 litres per minute. The liquid in tank 2 is pumped back into tank 1 at 1 litre per minute. In addition, pure water flows into the first tank at 3 litres per minute, and liquid drains out of the second tank at 3 litres per minute. Assume the mixture in each tank is well-stirred. Find the amount of salt in each tank after t minutes.

[10pts] 4. Solve the initial value problem:

$$y_1' = 4y_1 - 2y_2 - 14, \quad y_1(0) = 7$$
$$y_2' = 3y_1 - y_2 - 2x - 7, \quad y_2(0) = 3$$

[20pts] 5. Let $f(x) = \dfrac{2x}{1+x^2}$. This question is about numerical estimation of $\displaystyle\int_0^3 f(x)\,dx$.

(a) Calculate the integral exactly.

(b) Use Simpson's Rule with $2m = n = 6$ subintervals to approximate the integral, giving your answer to 6 decimal places. Compare your result with the true value by calculating the error ϵ_6.

(c) Approximately what would you expect the error to be if you used $n = 12$ subintervals? Estimate this without calculating or estimating any derivatives.

(d) Knowing that $f^{(4)}(x) < 50$ for all $0 < x < 3$, how big would n have to be to guarantee that Simpson's Rule will give an answer accurate to within 10^{-5}?

(e) Approximate the same integral by applying the two-point Gaussian Quadrature formula over each of the subintervals [1,2] and [2, 3]. Calculate the error.
(*See Table 9.3 and Example 6.11 in the printed course notes (DV).*)