MAT 2384 A Assignment 3

Winter 2019, Dr. Schmah

due Wednesday, February 27th, 4pm, in class

Students are encouraged to discuss homework problems with others. However, once you understand how to solve a problem, you must write out your own solution, without copying others' work.

In Questions 1 to 9, solve the initial value problem:

1. $x^2y'' - 4xy' + 6y = 0, \quad y(1) = 4, \quad y'(1) = 10$
2. $x^2y'' - 5xy' + 13y = 0, \quad x > 0, \quad y(1) = 0, \quad y'(1) = 6$
3. $y''' - 6y'' + 11y' - 6y = 0, \quad y(0) = 6, \quad y'(0) = 14, \quad y''(0) = 36$
4. $x^3y''' - x^2y'' + 2xy' - 2y = 0, \quad x > 0, \quad y(1) = -3, \quad y'(1) = -7, \quad y''(1) = -9$
5. $y'' - 4y = 12e^{2x} + 8x - 16, \quad y(0) = 10, \quad y'(0) = 1$
6. $y''' - 4y'' + y' + 6y = 10\cos x + 18x + 3, \quad y(0) = 5, \quad y'(0) = 17, \quad y''(0) = 33$
7. $y'' + 9y = \csc 3x, \quad y(\frac{\pi}{2}) = 1, \quad y'(\frac{\pi}{2}) = 0$
8. $x^2y'' - 2xy' + 2y = x^2, \quad x > 0, \quad y(1) = 4, \quad y'(1) = 5$
9. $y'' - 4y' + 4y = 3x^{-2}e^{2x}, \quad y(1) = y'(1) = 2e^2$
10. Consider the following data points (x_j, f_j), for $j = 0, 1, 2, 3$:
    $(0.2, 0.4651)$, $(0.4, 0.7336)$, $(0.75, 0.3532)$ and $(1.2, 1.6729)$. These are considered as points on the graph of an unknown function $f(x)$.
    (a) Find the third-degree interpolating polynomial $p_3(x)$ using Newton's Divided Differences, with coefficients to 4 decimal places.
    (b) Interpolate the value of f at $x = 1$.
    (c) Given that $1.2 \leq f^{(4)}(x) \leq 273$ on $[0.2, 1.2]$, find upper and lower bounds on the absolute error of the estimate in part (b).