MAT 2384 A Assignment 2

Winter 2019, Dr. Schmah

due Wednesday, February 6th, 4pm, in class

Students are encouraged to discuss homework problems with others. However, once you understand how to solve a problem, you must write out your own solution, without copying others' work.

In Questions 1 to 8, solve the initial value problem:

1. $(12xy^2 - 21\ln y + 5x^2)\,dx + (6x^2y - 7x/y)\,dy = 0, \quad y(1) = 1$
2. $(-2y\sin x + 2xy^3 - e^x y^2)\,dx + (4\cos x + 4x^2y^2 - 3e^x y)\,dy = 0, \quad y(0) = 2$
3. $y' + \dfrac{2}{x}\,y = x, \quad y(4) = 3$
4. $y' - 2y = 3e^x, \quad y(0) = 4$
5. $y' + y\,\tan x = y^2, \quad y(0) = 1/2$
6. $y'' - 9y = 0, \quad y(0) = 2, \quad y'(0) = 0$
7. $y'' + \pi^2 y = 0, \quad y(0) = 1, \quad y'(0) = \pi$
8. $y'' + 4y' + 4y = 0, \quad y(0) = 7, \quad y'(0) = -18$
9. Play with the Newton's method demonstration at:
   `https://demonstrations.wolfram.com/LearningNewtonsMethod/`
   (a) Using this tool (or another, or your own calculations) calculate 6 steps of Newton's method to for $f(x) = \cos x$ and $x_0 = 3.51$. Hand in these values, either hand-written or on a screenshot. Note that the closest root to x_0 is $3\pi/2 \approx 4.71$, but the iterations do not converge to this value.
   (b) Using the same f and x_0, and starting with initial bracketing interval $[\pi, 2\pi]$, calculate 4 steps of the global Newton bisection method.
10. Consider the following data points (x_j, f_j), for $j = 0, 1, 2, 3$:
    $(1.1, 3.2)$, $(1.6, 5.5)$, $(2.2, 8.0)$ and $(2.5, 9.1)$.
    (a) Find the Lagrange interpolating polynomial $p_3(x)$, with coefficients to 4 decimal places.
    (b) Interpolate values at $x = 1.25$ and $x = 2.0$.
    (c) Given that $0.5 \le f^{(4)}(x) \le 3.0$ on $[1, 3]$, estimate the error bounds for the two values in part (b).