MAT 2384 A Assignment 1

Winter 2019, Dr. Schmah

due Wednesday, January 23rd, in class

Students are encouraged to discuss homework problems with others. However, once you understand a solution, you must write it out entirely by yourself.

In Questions 1 to 5, solve the initial value problem:

1. $\cot x \, \sin y \, y' = 1, \quad y(0) = 0$
2. $xy' = y + x \sec(y/x), \quad y(1) = \pi/2$
3. $(\cos y + y^2 \cos x + 1)\, dx + (2y \sin x - x \sin y)\, dy = 0, \quad y(\pi) = \pi$
4. $(4xy^4 + 3y^2)\, dx + (8x^2y^3 + 6xy + 3y^2)\, dy = 0, \quad y(1) = -1$
5. $(x \ln(y^4) + 4 \ln y - y^2)\, dx + \left(-2y + \frac{4x}{y}\right) dy = 0, \quad y(0) = 1.$
6. If wet laundry loses half of its moisture in the first 10 minutes of drying in a dryer, and if the rate of loss of moisture is proportional to the remaining moisture, when will it have lost 90% of its moisture? Model this problem using an ODE, and then solve it.
7. Consider $f(x) = x^3 - 4x + 2$.
   (a) Prove that $f(x) = x^3 - 4x + 2$ has a root in $[0, 1]$.
   (b) Define a function $g(x)$ such that x is a fixed point of g if and only if it is a zero of f.
   (c) Verify that fixed-point iteration with your function g and $x_0 = 0.5$ will converge.
   (d) Starting with $x_0 = 0.5$, perform as many iterations as required to find a root of f to 6 decimal places.
8. Use Newton's Method to find the first positive solution of $e^{-x} = \sin x$ to 6 decimal places. Start with $x_0 = \pi/4$.