Gaussian Integration Handout

To convert $\int_a^b f(x)dx$ into $\int_{-1}^1 g(t)dt$, let $x = \frac{a+b}{2} + \frac{b-a}{2}t$. Then

$$\int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^{1} f\left(\frac{a+b}{2} + \left(\frac{b-a}{2}\right)t\right) dt.$$

Gaussian Quadrature of order n:

$$\int_a^b f(x)dx \sim \sum_{i=1}^{n} w_i f(t_i)$$

where the w_i's are the coefficients and the t_i's are the nodes (these are the roots of the Lagrange Polynomials $P_n(x)$). The method of Gaussian Quadrature of order n is **Exact** for Polynomials of degree $2n-1$ or less.

Order	Nodes	Coefficient
n	t_i	w_i
2	-0.5773502692	1.0
	0.5773502692	1.0
3	-0.7745966692	0.555555556
	0.0	0.888888889
	0.7745966692	0.555555556
4	-0.8611363116	0.3478548451
	-0.3399810436	0.6521451549
	0.3399810436	0.6521451549
	0.8611363116	0.3478548451
5	-0.9061798459	0.2369268850
	-0.5384693101	0.4786286705
	0.0	0.5688888889
	0.5384693101	0.4786286705
	0.9061798459	0.2369268850