University of Ottawa

Department of Mathematics and Statistics

MAT 2384C: Ordinary Differential Equations and Numerical Methods

Professor: Tanya Schmah

Final

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Last name ______________________ First name ______________________

Student number ______________________

Instructions:

- The duration of the exam is 3 hours.
- This is a closed book exam consisting of 9 questions, for a total of 60 points.
- There are 16 pages in this exam question booklet, of which pages 15 and 16 contain a table of formulas. Pages 14–16 may be detached if you wish and used for scrap paper.
- Answer the questions in this exam question booklet in the space provided. If you need extra space, you may use the backs of the pages as long as you clearly indicate where your answers are and the order in which different parts should be read (if needed). Otherwise, writing on the backs of pages will not be considered.
- Only **non-programmable, non-graphing calculators** are permitted.
- Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, academic fraud allegations may be filed, which may result in you obtaining a 0 (zero) for the exam, amongst other sanctions.

By signing below, you agree that you are complying with the above statement.

Signature ______________________

Good luck!

Do not write anything in the following table.

Question	1	2	3	4	5	6	7	8	9	Total
Maximum	7	7	7	6	9	4	8	6	6	60
Score										

Table of Formulas

$f(t)$	$F(s)=\mathcal{L}\{f(t)\}$	
t^n	$n!/s^{n+1}$	; $n=0,1,2,\dots$ and $s>0$
e^{at}	$1/(s-a)$	; $s>a$
$\sin(kt)$	$k/(s^2+k^2)$	; $s>0$
$\cos(kt)$	$s/(s^2+k^2)$	; $s>0$
$\delta(t-a)$	e^{-as}	; $s>0$
$u(t-a)$	$\frac{e^{-as}}{s}$	; $s>0$

$$\begin{aligned}
\mathcal{L}\{f(t)\}(s) &= \int_0^\infty e^{-st}f(t)dt \\
\mathcal{L}\{e^{at}f(t)\} &= F(s-a) \\
\mathcal{L}\{u(t-a)f(t-a)\} &= e^{-as}F(s) \\
\mathcal{L}\{t^n f(t)\} &= (-1)^n\frac{d^n}{ds^n}F(s) \\
\mathcal{L}\left\{\int_0^t f(x)dx\right\} &= \frac{1}{s}F(s) \\
\mathcal{L}\left\{\frac{f(t)}{t}\right\} &= \int_s^\infty F(x)dx \\
\mathcal{L}\left\{\frac{d^n}{dt^n}f(t)\right\} &= s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)-\ldots-f^{(n-1)}(0) \\
(f*g)(t) &= \int_0^t f(x)g(t-x)dx \\
\mathcal{L}\{(f*g)(t)\} &= F(s)\,G(s)
\end{aligned}$$

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

$$p_n(x)=L_0(x)f_0+\cdots+L_n(x)f_n$$

$$L_i(x)=\frac{(x-x_0)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$$

$$\epsilon_n(x)=f(x)-p_n(x)=(x-x_0)\cdots(x-x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!}$$

$$p_n(x)=f_0+f[x_0,x_1](x-x_0)+f[x_0,x_1,x_2](x-x_0)(x-x_1)+\ldots$$
$$\cdots+f[x_0,\ldots x_n](x-x_0)\ldots(x-x_{n-1})$$

$$f[x_j,x_{j+1}]=\frac{f_{j+1}-f_j}{x_{j+1}-x_j}$$

$$\vdots$$

$$f[x_j,\ldots x_k]=\frac{f[x_{j+1},\ldots,x_k]-f[x_j,x_{k-1}]}{x_k-x_j}$$

$$\int_a^b f(x)dx = h\sum_{j=1}^{n} f(x_j^*)\,,\ |\epsilon| \le \frac{M(b-a)^3}{24n^2}\,,\ M = max_{a\le x\le b}\,|f''(x)|$$

$$\int_a^b f(x)dx = \frac{h}{2}\sum_{j=1}^{n} \left(f(x_{j-1}) + f(x_j)\right)\,,\ |\epsilon| \le \frac{M(b-a)^3}{12n^2}\,,\ M = max_{a\le x\le b}\,|f''(x)|$$

$$\int_a^b f(x)dx = \frac{h}{3}\sum_{j=0}^{n-1} \left(f(x_{2j}) + 4f(x_{2j+1}) + f(x_{2j+2})\right)\,,\ |\epsilon| \le \frac{M(b-a)^5}{180n^4}\,,$$
$$M = max_{a\le x\le b}\,|f^{(4)}(x)|$$

$$x = \tfrac{b+a}{2} + \tfrac{b-a}{2}t$$
$$\textstyle\int_{-1}^{1} g(t)dt \approx w_1 g(t_1) + \cdots + w_n g(t_n)$$

Ordre	Noeuds	Coefficients
n	t_i	w_i
2	-0.5773502692	1.0
	0.5773502692	1.0
3	-0.7745966692	0.555555556
	0.0	0.888888889
	0.7745966692	0.555555556
4	-0.8611363116	0.3478548451
	-0.3399810436	0.6521451549
	0.3399810436	0.6521451549
	0.8611363116	0.3478548451
5	-0.9061798459	0.2369268850
	-0.5384693101	0.4786286705
	0.0	0.5688888889
	0.5384693101	0.4786286705
	0.9061798459	0.2369268850

$$\begin{aligned} x_{n+1} &= x_n + h \\ y_{n+1}^P &= y_n^C + h\,f(x_n, y_n^C) \\ y_{n+1}^C &= y_n^C + \frac{1}{2}h\left(f(x_n, y_n^C) + f(x_{n+1}, y_{n+1}^P)\right) \end{aligned}$$

$$\begin{aligned} x_{n+1} &= x_n + h \\ k_1 &= h\,f(x_n, y_n) \\ k_2 &= h\,f(x_n + \frac{1}{2}\,h, y_n + \frac{1}{2}\,k_1) \\ k_3 &= h\,f(x_n + \frac{1}{2}\,h, y_n + \frac{1}{2}\,k_2) \\ k_4 &= h\,f(x_n + h, y_n + k_3) \\ y_{n+1} &= y_n + \frac{1}{6}\,(k_1 + 2\,k_2 + 2\,k_3 + k_4) \end{aligned}$$