MAT 1348A (Prof. T. Schmah) — Fourth Homework Assignment
Due Thursday Feb. 25, 2016 by 11:00am (NEW due time)

Instructions:

Print out this document and staple the pages. You may write on both sides of the paper or insert additional pages if necessary.

Submit a finished, presentable product. *Drafts and illegible papers will not be marked.* Show all relevant work to receive full credit.

Submit the assignment to your TA in the DGD or in the *submission box labeled MAT 1348A* in the Department of Mathematics and Statistics.

Circle the DGD you attend. Your marked paper will be returned to you in that DGD.

Late assignments will not be accepted.

Important note on academic integrity:

Students are permitted, and indeed encouraged, to discuss homework problems with others, but are not permitted to help each other write the final solutions (unless the assignment is explicitly announced as a group assignment). Once you understand a solution, you must write it out entirely by yourself. For each question, any help from other people must be clearly acknowledged, as well as any sources used (e.g. textbooks, websites, videos), if that source contains a solution to a very similar question, or a new method or idea that you used that was not in the course materials. Failure to follow these rules constitutes plagiarism (academic fraud). Note that if one student copies from the other, both students have committed academic fraud. If we believe plagiarism has occurred, the students will receive:

- a mark of 0 for the current assignment if this is the first offence;

- a mark of 0 for the whole assignment component of the course if this is the second offence.

Students are advised to carefully examine the *University Guidelines on Academic Integrity* — see

> http://web5.uottawa.ca/mcs-smc/academicintegrity/home.php

as well as the *Course Policy on Plagiarism* — see

> http://mysite.science.uottawa.ca/msajna//teaching/plagiarism_policy.html

Please sign below to confirm that you have read and understood these regulations and policies, and you agree to act with academic integrity as defined therein.

Student's signature:

1. Give a **proof by contradiction** of the following theorem:

   Let n be an integer. Then at least one of the integers n and $n + 1348$ is not divisible by 3. [4pts]

2. A spy (who can lie or tell the truth) arrives on the Island of Knights and Knaves. You encounter three people, A, B, and C, and you know that one is a knight, one a knave, and one a spy. Each of these three knows the type of the other two.

A says: "I am the knight."

B says: "A is not the knave."

C says: "B is not the knave."

Using a **proof by cases**, determine who is the knight, who is the knave, and who is the spy. [4pts]

3. Let A, B, C be three subsets of a universal set $\mathcal{U}$. The **symmetric difference** of sets A and B is

$$A \oplus B = \{x \in \mathcal{U} : x \in A \oplus x \in B\}$$

(also denoted $A \Delta B$), where the $\oplus$ on the right hand side means "exclusive or".

For each of the following two statements, determine whether it is **true** or **false**. If you claim that the statement is true, **prove** it; otherwise, give a **counterexample**. [5pts]

(a) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$ for all A, B, C.

(b) For all A, B, C, if $A \cup B = A \cup C$, then $B = C$.