Circle: DGD 1 (Frédéric)
DGD 2 (Yue)
DGD 3 (Andrew)

SOLUTIONS

Marks: /14

Student number:

MAT 1348A (Prof. T. Schmah) — Second Homework Assignment
Due Jan. 27, 2016 by 3:00pm

Instructions:

Print out this document and staple the pages. You may write on both sides of the paper or insert additional pages if necessary.

Submit a finished, presentable product. *Drafts and illegible papers will not be marked.* Show all relevant work to receive full credit.

Submit the assignment to your TA in the DGD or in the *submission box labeled MAT 1348C* in the Department of Mathematics and Statistics.

Circle the DGD you attend. Your marked paper will be returned to you in that DGD.

Late assignments will not be accepted.

Important note on academic integrity:

Students are permitted, and indeed encouraged, to discuss homework problems with others, but are not permitted to help each other write the final solutions (unless the assignment is explicitly announced as a group assignment). Once you understand a solution, you must write it out entirely by yourself. For each question, any help from other people must be clearly acknowledged, as well as any sources used (e.g. textbooks, websites, videos), if that source contains a solution to a very similar question, or a new method or idea that you used that was not in the course materials. Failure to follow these rules constitutes plagiarism (academic fraud). Note that if one student copies from the other, both students have committed academic fraud. If we believe plagiarism has occurred, the students will receive:

- a mark of 0 for the current assignment if this is the first offence;
- a mark of 0 for the whole assignment component of the course if this is the second offence.

Students are advised to carefully examine the *University Guidelines on Academic Integrity* — see

`http://web5.uottawa.ca/mcs-smc/academicintegrity/home.php`

as well as the *Course Policy on Plagiarism* — see

`http://mysite.science.uottawa.ca/msajna//teaching/plagiarism_policy.html`

Please sign below to confirm that you have read and understood these regulations and policies, and you agree to act with academic integrity as defined therein.

Student's signature:

1. The table below is a truth table for a mystery compound proposition P, containing propositional variables a, b, and c, and logical connectives. [5pts]

a	b	c	P
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T ←
F	T	F	F
F	F	T	T ←
F	F	F	T ←

[1] (a) Write a **DNF formula** (Disjunctive Normal Form) for P (without simplifying).

[3] (b) Using the **table of logical equivalences**, find a proposition Q logically equivalent to P that is as simple as possible (that is, it has as few terms as possible).

[1] (c) Find a compound proposition R logically equivalent to P that contains **only logical connectives** $\neg$ **and** $\wedge$.

a) DNF for P: $(\neg a \wedge b \wedge c) \vee (\neg a \wedge \neg b \wedge c) \vee (\neg a \wedge \neg b \wedge \neg c)$

b) $(\neg a \wedge b \wedge c) \vee (\underline{\neg a \wedge \neg b} \wedge c) \vee (\underline{\neg a \wedge \neg b} \wedge \neg c)$

$\equiv (\neg a \wedge b \wedge c) \vee ((\neg a \wedge \neg b) \wedge \underline{(c \vee \neg c)})$ (Distributive)

$\equiv (\neg a \wedge b \wedge c) \vee ((\neg a \wedge \neg b) \wedge \mathbf{T})$ (Negation)

$\equiv (\underline{\neg a} \wedge b \wedge c) \vee (\underline{\neg a} \wedge \neg b)$ (Identity)

$\equiv \neg a \wedge ((b \wedge c) \vee \neg b)$ (Dist.)

$\equiv \neg a \wedge (\underline{(b \vee \neg b)} \wedge (c \vee \neg b))$ (Comm. + Dist.)

$\equiv \neg a \wedge (\mathbf{T} \wedge (c \vee \neg b))$ (Negation)

$\equiv \neg a \wedge (\neg b \vee c)$ (Id. & Comm.)

c) $P \equiv \neg a \wedge (\neg b \vee c)$

$\equiv \neg a \wedge (\neg (b \wedge \neg c))$ (De Morgan)

$R = \neg a \wedge (\neg (b \wedge \neg c))$

Comments on 1 b)

- There are other ways to get to the same answer, see eg Solutions on 1348C webpage

- Though the question asked you to use the table of logical equivalences, another method would have been to look at the truth table and group the rows:

$$\begin{array}{l} \text{last 2} \\ \text{rows} \end{array} \left\{ \begin{array}{ccccc} F & F & T & T & \leftarrow \\ F & F & F & T & \leftarrow \end{array} \right.$$

p is true when $a = b = F$, regardless of the value of c. So you could immediately write a DNF:

$$\underbrace{(\neg a \wedge b \wedge c)}_{\text{row 5}} \vee \underbrace{(\neg a \wedge \neg b)}_{\text{rows 7\&8}}$$

Alternatively, note that all solutions have $a = F$, then just consider the sub-table for b and c when $a = F$:

$$\begin{array}{lccc} & T & T & \leftarrow \\ & T & F & \\ \text{group} \left\{ \begin{array}{l} \\ \\ \end{array} \right. & \begin{array}{c} F \\ F \end{array} & \begin{array}{c} T \\ F \end{array} & \begin{array}{c} \leftarrow \\ \leftarrow \end{array} \end{array}$$

Then you can immediately write $\neg a \wedge ((b \wedge c) \vee \neg b)$

2. Use **truth tables** to answer this question. Consider the following three compound propositions in propositional variables x and y: [5pts]

P: $(x \leftrightarrow y) \rightarrow (x \vee y)$
Q: $(x \oplus y) \wedge \neg(x \rightarrow \neg y)$
R: $\neg(x \wedge y) \wedge (x \leftrightarrow y)$

[3] (a) Complete a truth table for P, Q, and R.

[1] (b) For each of P, $\neg P$, Q, $\neg Q$, R, and $\neg R$, determine whether it is a tautology, a contradiction, or a contingency.

[1] (c) Which propositions among $P, \neg P, Q, \neg Q, R, \neg R$ are **logically equivalent**?

(a)

x	y	$x \leftrightarrow y$	$x \vee y$	$x \oplus y$	$x \rightarrow \neg y$	$x \wedge y$	P	Q	R
T	T	T	T	F	F	T	T	F	F
T	F	F	T	T	T	F	T	F	F
F	T	F	T	T	T	F	T	F	F
F	F	T	F	F	T	F	F	F	T

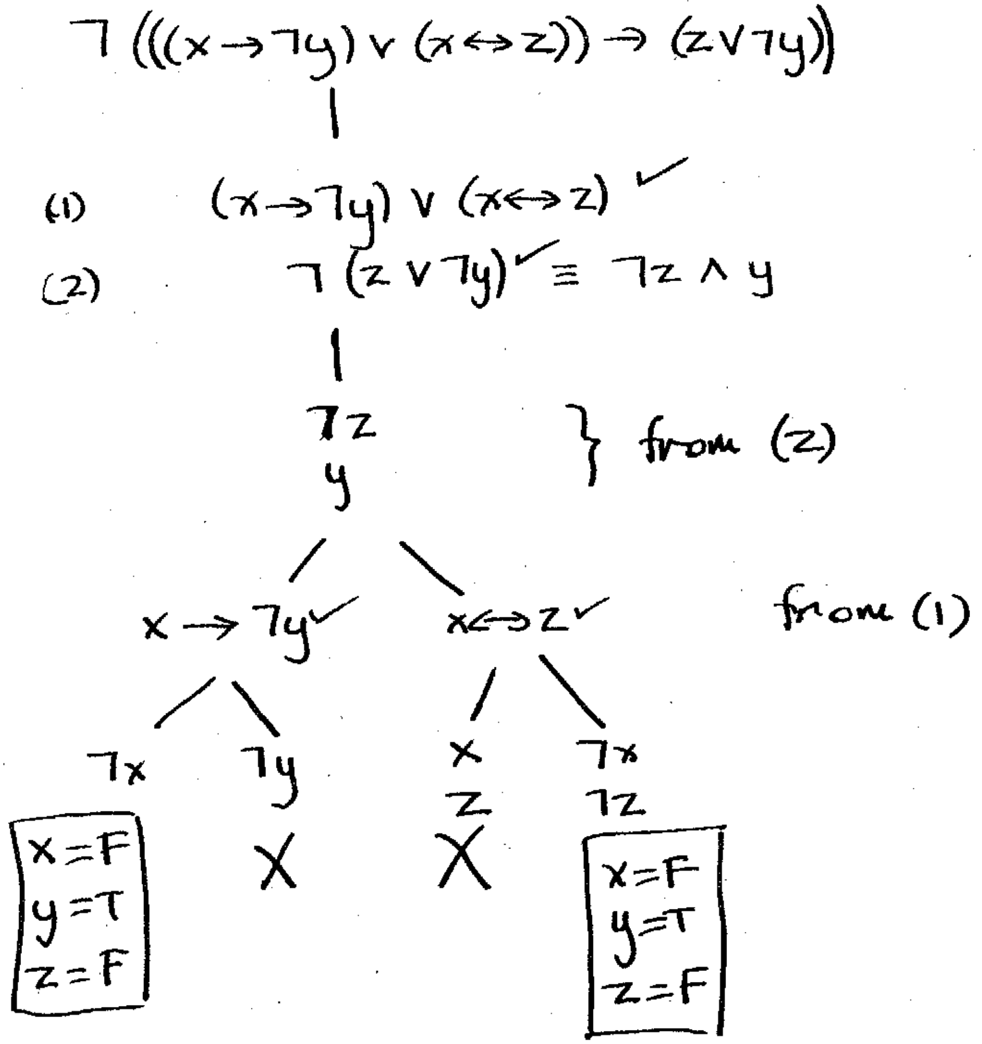
(b) Tautologies : ¬Q
Contradictions : Q
Contingencies : P, ¬P, R, ¬R

(c) P ≡ ¬R
and ¬P ≡ R

3. Let P be the compound proposition

$$\Big((x \to \neg y) \vee (x \leftrightarrow z)\Big) \to (z \vee \neg y).$$

Using an appropriate **truth tree**, determine whether or not P is a **tautology**. If you claim that it is not, give **all counterexamples**. [4pts]



Conclusion: since there are complete open branches, $\neg P$ is not a contradiction, so P is not a tautology. There is only one counter-example: $x = F$, $y = T$, $z = F$.