# Mathematical modelling of clonorchiasis with human treatment and fish vaccination versus snail control 

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#### Abstract

Clonorchiasis is an important food-borne parasitic disease that has been largely understudied. We examine the dynamics of clonorchiasis with human treatment and fish vaccination versus snail control. Four mathematical models are proposed and analysed, both analytically and numerically. We show that fish vaccination is a valid method of control and derive the maximal period of molluscicide application in order to control the snail populations. It follows that a variety of methods may be necessary to control clonorchiasis.


## 1. Introduction

The liver fluke Clonorchis sinensis is a high-risk pathogenic parasitic helminth that is predominantly endemic in Asian countries including Korea, China, Taiwan, Vietnam and the far eastern parts of Russia - and is still actively transmitted, causing the disease condition clonorchiasis, which is associated with urinary bladder cancer [1]. Currently, more than 35 million people are infected with clonorchiasis [2], with 750 million at risk [3]. The public-health importance of liver-fluke infections, especially clonorchiasis, has been neglected for decades, despite a persistent and growing number of infections and corresponding disease burden [4].

Snails act as a first intermediate host, while freshwater fish act as a secondary intermediate host [4]. Freshwater snails ingest eggs laid by hermaphrodite adult worms and produce cercariae, which escape from the snails and adhere to freshwater fish [5]. When people eat raw or undercooked fish, metacercarie separate from the flesh through gastric digestion, and juvenile flukes migrate to the bile ducts, where they become adult flukes [6]. Eggs laid by hermaphrodite adult worms reach the human intestine and are eliminated with the faeces [7]. The egg-laying capacity of an adult $C$. sinensis fluke has been estimated at around 4000 eggs per worm per day [8].

Piscivorous animals, especially cats and dogs (both wild or reared as pets or guardians), can also serve as reservoir hosts for C. sinensis; these animals are widely distributed and can maintain the lifecycle of the parasite in endemic areas without involvement of people [4]. Infection
in cats is higher than dogs, possibly due to their preference for eating fish [9].

Praziquantel is virtually the only drug for treating C. sinensis infections and has been recommended by the World Health Organization (WHO) for more than 30 years [10]. Monotherapy is $70 \%-90 \%$ effective, but this is a potentially dangerous situation if drug resistance should emerge [11]; possible emergence of praziquantel-resistant strains (or isolates) of S. mansoni and S. japonicum has been documented in the laboratory [12], raising the spectre that resistance may emerge against $C$. sinensis. Another drug, Albendazole, requires long treatment courses over multiple days [13] and has a high rate of egg reduction but low cure rates [14] and so is rarely used [11]. Reinfection in animals is common, making practical treatment of cats and dogs infeasible [15].

No commercially produced or effective vaccine is available for the treatment of $C$. sinensis infection in human or other hosts as of yet. However, the possibility of developing a fish vaccine, in combination with the use of non-polluted water for the culture of fish, has been proposed [16]. Snail control has also been suggested as a way to interrupt the transmission cycle [17], specifically with predator fish [4], as widespread molluscicide is not recommended [18]. Other environmental controls include removing toilets over fish ponds although complete removal of faecal contamination is not feasible [4] - or culturally sensitive education aimed at raising awareness of the transmission cycle and stimulating behaviour changes that discourage consumption of raw fish and improve sanitary practices [18]. The

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Fig. 1. Schematic diagram of model (2.1) of clonorchiasis with human treatment and fish vaccination versus snail control.
cost-effectiveness of intervention measures in China has been examined for chemotherapy, education and environmental control, with chemotherapy proving to be the most cost-effective strategy [19].

Only a handful of models have been developed for clonorchiasis. Song and Kang [20] used a mathematical analysis of age prevalence to determine egg-positive rates of clonorchiasis and developed catalytic curves for two study regions in Korea. Keiser and Utzinger [21] used a random-effects model to determine relative risk calculations for several studies of foodborne trematodiases, including C. sinensis, in order to determine the effect of the proximity to freshwater bodies. They showed that residents living near fresh water have a 2.15 -fold higher risk for infections compared to people living further from the water. Qian et al. [22] used Monte Carlo simulations to determine overall disability weights for $C$. sinensis infection. They showed that the disability weights differed by sex and age and determined a correlation between disability and infection intensity. Lai [23] developed Bayesian geostatistical logistic regression models with location-specific random effects to obtain spatially explicit $C$. sinensis risk estimates for China in order to identify areas of high priority for control. Yuan et al. [17] developed an SEIR model for humans, with susceptible and infected snails and fish, along with C. sinensis eggs and cercariae, with predation of fish. They found the reproduction number and used data from China and a sensitivity analysis to show that drug treatment of humans alone would not be enough to control C. sinensis. Gao et al. [24] developed a model that assessed intervention techniques of snail control, health education and chemotherapy in China. They found that strengthening health education and improving faeces management would make moderate gains, but the most important tool was snail control. Zhang et al. [25] presented a model of clonorchiasis that included three time delays and found that breaking the life cycle of Clonorchis sinensis would be more effective than treating the disease.

Here, we introduce several mathematical models, which we develop in order to consider possible biological scenarios. Our main purpose is to investigate biological factors that influence and control invasion, persistence and variability of the disease in natural ecosystems by comparing their dynamics.

## 2. Model I: The baseline model

First, we consider fish $F_{i}$, humans $H_{i}$ and infected molluscs $M$, with the worms, eggs and parasites not explicitly incorporated in the model. We assume that susceptible fish $F_{s}$ grow logistically, since the infection is not passed vertically. Transmission between species is described by a mass-action process, as we can assume there is no selectivity and that populations are well-mixed.

Humans will consume both susceptible and infected fish, $F_{i}$, reducing each of their populations, while we add in a linear death rate to the infected classes. Susceptible humans can become infected upon eating infected fish, while the recruitment rate of infected molluscs is proportional to the number of infected humans. A schematic diagram is provided by Fig. 1. Note that $\delta$ represents both the natural and the disease-related mortality in humans.
$\frac{d F_{s}}{d t}=r_{1}\left(F_{s}+F_{i}\right)\left(1-\frac{F_{s}+F_{i}}{k_{1}}\right)-\theta_{1} F_{s}\left(H_{s}+H_{i}\right)-\gamma F_{s} M$,
$\frac{d F_{i}}{d t}=\gamma F_{s} M-\theta_{1} F_{i}\left(H_{s}+H_{i}\right)-\epsilon F_{i}$,
$\frac{d H_{s}}{d t}=r_{2}\left(H_{s}+H_{i}\right)-\alpha \theta_{1} F_{i} H_{s}$,
$\frac{d H_{i}}{d t}=\alpha \theta_{1} F_{i} H_{s}-\delta H_{i}$,
$\frac{d M}{d t}=\beta H_{i}-\mu M$.
First, we examine the local stability of system (2.1). We write $J_{k}^{I}=J$ evaluated at $E_{k}^{I}, k=0,1,2$. System (2.1) has three steady states: $E_{0}, E_{1}$ and $E_{2}$, where

$$
\begin{aligned}
E_{0}^{I}:\left(M, F_{i}, F_{s}, H_{i}, H_{s}\right)= & (0,0,0,0,0), \\
E_{1}^{I}:\left(M, F_{i}, F_{s}, H_{i}, H_{s}\right)= & \left(0,0, k_{1}, 0,0\right), \\
E_{2}^{I}:\left(M, F_{i}, F_{s}, H_{i}, H_{s}\right)= & \left(\frac{\beta r_{2} R_{1}}{\left(\delta-r_{2}\right) \mu}, \frac{\delta r_{2}}{\alpha \theta_{1}\left(\delta-r_{2}\right)},\right. \\
& \left.\frac{\delta \mu\left(\delta R_{1} \theta_{1}+\delta \epsilon-\epsilon r_{2}\right)}{\alpha \beta \gamma R_{1} \theta_{1}\left(\delta-r_{2}\right)}, \frac{r_{2} R_{1}}{\delta-r_{2}}, R_{1}\right),
\end{aligned}
$$

where $R_{1}$ is a positive root of the equation $\sum_{0}^{3} Y_{i} Z^{i}=0$, with $Y_{i}$ given by

$$
\begin{align*}
Y_{3}= & \alpha \beta^{2} \delta \gamma^{2} k_{1} r_{2} \theta_{1}^{2}+\alpha \beta \delta^{2} \gamma \mu k_{1} \theta_{1}^{3},  \tag{2.2a}\\
Y_{2}= & \left(\alpha \beta^{2} \delta \epsilon \gamma^{2} k_{1} r_{2} \theta_{1}-\alpha \beta^{2} \delta \gamma^{2} k_{1} r_{1} r_{2} \theta_{1}-\alpha \beta^{2} \epsilon \gamma^{2} k_{1} r_{2}^{2} \theta_{1}\right. \\
& +\alpha \beta^{2} \gamma^{2} k_{1} r_{1} r_{2}^{2} \theta_{1}+\alpha \beta \delta^{2} \epsilon \gamma \mu k_{1} \theta_{1}^{2}+\alpha \beta \delta^{2} \gamma \mu \psi k_{1} \theta_{1}^{2} \\
& -\alpha \beta \delta^{2} \gamma \mu k_{1} r_{1} \theta_{1}^{2}-\alpha \beta \delta \epsilon \gamma \mu k_{1} r_{2} \theta_{1}^{2}, \\
& -\alpha \beta \delta \gamma \mu \psi k_{1} r_{2} \theta_{1}^{2}+\alpha \beta \delta \gamma \mu k_{1} r_{1} r_{2} \theta_{1}^{2}+\beta^{2} \delta \gamma^{2} r_{1} r_{2}^{2} \\
& \left.+2 \beta \delta^{2} \gamma \mu r_{1} r_{2} \theta_{1}+\delta^{3} \mu^{2} r_{1} \theta_{1}^{2}\right),  \tag{2.2b}\\
Y_{1}= & \left(\alpha \beta \delta^{2} \epsilon \gamma \mu \psi k_{1} \theta_{1}-\alpha \beta \delta^{2} \epsilon \gamma \mu k_{1} r_{1} \theta_{1}-2 \alpha \beta \delta \epsilon \gamma \mu \psi k_{1} r_{2} \theta_{1},\right. \\
& +2 \alpha \beta \delta \epsilon \gamma \mu k_{1} r_{1} r_{2} \theta_{1}+\alpha \beta \epsilon \gamma \mu \psi k_{1} r_{2}^{2} \theta_{1}-\alpha \beta \epsilon \gamma \mu k_{1} r_{1} r_{2}^{2} \theta_{1}, \\
& \left.+2 \beta \delta^{2} \epsilon \gamma \mu r_{1} r_{2}-2 \beta \delta \epsilon \gamma r_{1} r_{2}^{2}+2 \delta^{3} \epsilon \mu^{2} r_{1} \theta_{1}-2 \delta^{2} \epsilon \mu^{2} r_{1} r_{2} \theta_{1}\right),  \tag{2.2c}\\
Y_{0}= & \delta^{3} \epsilon^{2} \mu^{2} r_{1}-2 \delta^{2} \epsilon^{2} \mu^{2} r_{1} r_{2}+\delta \epsilon^{2} \mu^{2} r_{1} r_{2}^{2} . \tag{2.2~d}
\end{align*}
$$

The equation $\sum_{0}^{3} Y_{i} Z^{i}=0$ has exactly one real positive root if $G^{2}+4 H^{3}>0$, it has two equal roots if $G^{2}+4 H^{3}=0$, and it has three distinct real roots if $G^{2}+4 H^{3}>0$, where $G=Y_{1}{ }^{2} Y_{4}-3 Y_{1} Y_{2} \Upsilon_{3}+2 Y_{2}{ }^{3}$, $H=Y_{1} Y_{3}-\Upsilon_{2}{ }^{2}$. Using Cardan's method, we obtain that the root is $\frac{1}{Y_{1}}\left(q-\frac{H}{q}-Y_{2}\right)$, where $q$ denotes one of the three values of $\left[\frac{1}{2}(-G+\right.$ $\left.\left.\sqrt{G^{2}+4 H^{3}}\right)^{\frac{1}{3}}\right]$. The condition $G^{2}+4 H^{3}=0$ yields two equal roots, and $G^{2}+4 H^{3}=0$ gives three distinct real roots; by Cardan's method, we can obtain the roots. In a subsequent section, we will obtain these roots using numerical simulations.

The general Jacobian matrix of system (2.1) is
$\boldsymbol{J}^{I}=\left[\begin{array}{ccccc}j_{11} & j_{12} & -\theta_{1} F_{s} & -\theta_{1} F_{s} & -\gamma F_{s} \\ \gamma M & -\theta_{1}\left(H_{s}+H_{i}\right)-\epsilon & -\theta_{1} F_{i} & -\theta_{1} F_{i} & \gamma F_{s} \\ 0 & -\alpha \theta_{1} H_{s} & j_{331} & j_{34} & 0 \\ 0 & \alpha \theta_{1} H_{s} & \alpha \theta_{1} F_{i} & -\delta & 0 \\ 0 & 0 & 0 & \beta & -\mu\end{array}\right]$,
where $j_{11}, j_{12}, j_{33}$ and $j_{34}$ are defined as follows:
$j_{11}=r_{1}\left(1-\frac{F_{s}+F_{i}}{k_{1}}\right)-\frac{r_{1}\left(F_{s}+F_{i}\right)}{k_{1}}-\theta_{1}\left(H_{s}+H_{i}\right)-\gamma M$,
$j_{12}=r_{1}\left(1-\frac{F_{s}+F_{i}}{k_{1}}\right)-\frac{r_{1}\left(F_{s}+F_{i}\right)}{k_{1}}$,
$j_{33}=r_{2}\left(1-\frac{H_{s}+H_{i}}{k_{2}}\right)-\frac{r_{2}\left(H_{s}+H_{i}\right)}{k_{2}}-\alpha \theta_{1} F_{i}$,
$j_{34}=r_{2}\left(1-\frac{H_{s}+H_{i}}{k_{2}}\right)-\frac{r_{2}\left(H_{s}+H_{i}\right)}{k_{2}}$.
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where $j_{11}, j_{12}, j_{33}$ and $j_{34}$ are defined as follows:
$j_{11}=r_{1}\left(1-\frac{F_{s}+F_{i}}{k_{1}}\right)-\frac{r_{1}\left(F_{s}+F_{i}\right)}{k_{1}}-\theta_{1}\left(H_{s}+H_{i}\right)-\gamma M$,
$j_{12}=r_{1}\left(1-\frac{F_{s}+F_{i}}{k_{1}}\right)-\frac{r_{1}\left(F_{s}+F_{i}\right)}{k_{1}}$,
$j_{33}=r_{2}\left(1-\frac{H_{s}+H_{i}}{k_{2}}\right)-\frac{r_{2}\left(H_{s}+H_{i}\right)}{k_{2}}-\alpha \theta_{1} F_{i}$,
$j_{34}=r_{2}\left(1-\frac{H_{s}+H_{i}}{k_{2}}\right)-\frac{r_{2}\left(H_{s}+H_{i}\right)}{k_{2}}$.
The Jacobian matrix at $E_{0}^{I}$ is given by
$J_{0}^{I}=\left[\begin{array}{ccccc}-\psi+r_{1} & r_{1} & 0 & 0 & 0 \\ 0 & -\epsilon & 0 & 0 & 0 \\ 0 & 0 & r_{2} & r_{2} & 0 \\ 0 & 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & \beta & -\mu\end{array}\right]$.
The eigenvalues of $J_{0}^{I}$ are $-\psi+r_{1},-\epsilon, r_{2},-\delta,-\mu$. Since $r_{2}>0, E_{0}$ is always unstable. The Jacobian matrix at $E_{1}^{I}$ is given by
$J_{1}^{I}=\left[\begin{array}{ccccc}\psi-r_{1} & 2 \psi-r_{1} & \frac{\theta_{1} k_{1}\left(\psi-r_{1}\right)}{r_{1}} & \frac{\theta_{1} k_{1}\left(\psi-r_{1}\right)}{r_{1}} & \frac{\gamma k_{1}\left(\psi-r_{1}\right)}{r_{1}} \\ 0 & -\epsilon & 0 & 0 & -\frac{\gamma k_{1}\left(\psi-r_{1}\right)}{r_{1}} \\ 0 & 0 & r_{2} & r_{2} & 0 \\ 0 & 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & \beta & -\mu\end{array}\right]$.

The eigenvalues of $J_{1}^{I}$ are $\psi-r_{1},-\epsilon, r_{2},-\delta,-\mu . E_{1}^{I}$ is unstable, since $r_{2}>0$. Note that the stability of $E_{2}^{I}$ can only be shown numerically.

Since $E_{0}$ is always unstable, extinction of all species is impossible. The details of the stability analysis shows that the extinction of all species is impossible due to the positive human net birth rate $r_{2}$. Finally, the stability of $E_{2}$ is conditional.

To illustrate our results, we ran several numerical simulations. Some parameter values in Table 1 are taken from the ecology literature, and some data are estimated. Fig. 2 shows the numerical solution of system (2.1). Solutions oscillate before reaching a coexistence equilibrium.

## 3. Model II: Fish vaccination and linear human growth

Here, we modify model (2.1) by considering vaccination of fish. See Fig. 3. Let $\psi$ be the rate of vaccination of susceptible fish and $F_{v}$ be the vaccinated fish population.

With these assumptions, model (2.1) reduces to
$\frac{d F_{s}}{d t}=r_{1}\left(F_{s}+F_{i}+F_{v}\right)\left(1-\frac{F_{s}+F_{i}+F_{v}}{k_{1}}\right)-\theta_{1} F_{s}\left(H_{s}+H_{i}\right)$

$$
\begin{equation*}
-\gamma F_{s} M-\psi F_{s}, \tag{3.1a}
\end{equation*}
$$

System (4.1) possesses the following equilibrium points:
$E_{0}^{I I}:\left(M, F_{i}, F_{s}, F_{v}, H_{i}, H_{s}\right)=(0,0,0,0,0,0)$,
$E_{1}^{I I}:\left(M, F_{i}, F_{s}, F_{v}, H_{i}, H_{s}\right)=\left(0,0, \frac{\left(-\psi \phi+\phi r_{1}+\psi r_{1}\right) k_{1} \phi}{\left(\phi^{2}+2 \psi \phi+\psi^{2}\right) r_{1}}\right.$,

$$
\left.\frac{\left(-\psi \phi+\phi r_{1}+\psi r_{1}\right) \psi k_{1}}{\left(\phi^{2}+2 \psi \phi+\psi^{2}\right) r_{1}}, 0,0\right)
$$

$E_{2}^{I I}:\left(M, F_{i}, F_{s}, F_{v}, H_{i}, H_{s}\right)$
$=\left(\frac{\beta r_{2} R_{2}}{\left(\delta-r_{2}\right) \mu}, \frac{r_{2}(\delta+\omega)}{\alpha \theta_{1}\left(\delta-r_{2}\right)}\right.$,
$\frac{\left(R_{2} \delta^{2} \theta_{1}+R_{2} \delta \omega \theta_{1}+\delta^{2} \epsilon+\delta \epsilon \omega-\delta \epsilon r_{2}-\epsilon \omega r_{2}\right) \mu}{\left(\delta-r_{2}\right) \theta_{1} R_{2} \gamma \alpha \beta}$,
$\frac{\mu \psi\left(R_{2} \delta^{2} \theta_{1}+R_{2} \delta \omega \theta_{1}+\delta^{2} \epsilon+\delta \epsilon \omega-\delta \epsilon r_{2}-\epsilon \omega r_{2}\right)}{\left(R_{2} \delta \theta_{1}+\delta \phi-r_{2} \phi\right) \gamma \alpha \beta R_{2} \theta_{1}}$,
$\left.\frac{r_{2} R_{2}}{\delta-r_{2}}, R_{2}\right)$,
where $R_{2}$ is a positive root of the equation $\sum_{0}^{5} Y_{i} Z^{i}=0$, with $Y_{i}$ as in Eqs. (2.2).

The equilibrium point $E_{0}^{I I}$ always exists. However, the equilibrium point $E_{1}^{I I}$ exists if and only if $r_{1}>r_{1}^{I I}$, where $r_{1}^{I I}=\frac{\psi \phi}{\psi+\phi}$; that is, the birth rate of the fish population is greater than the threshold $r_{1}^{I I}$. Note that the threshold increases with $\psi$, so fish vaccination is a valid method of control.

Table 1
Parameter values used in numerical simulations.

| Parameter | Description | Value (unit) | Reference |
| :--- | :--- | :--- | :--- |
| $r_{1}$ | Intrinsic growth rate of fish | 0.2 day $^{-1}$ | $[4,26]$ |
| $r_{2}$ | Intrinsic growth rate of humans | $0.01 \mathrm{day}^{-1}$ | $[4,27]$ |
| $k_{1}$ | Carrying capacity of fish | $1000 \mathrm{~m}^{-2}$ | $[17,26]$ |
| $k_{2}$ | Carrying capacity of humans | $500 \mathrm{~km}^{-2}$ | $[17,26]$ |
| $\gamma$ | Infection rate of fish | 0.0025 day $^{-1}$ | $[28,29]$ |
| $\theta_{1}$ | Consumption rate of fish by humans | 0.025 humans $^{-1}$ day $^{-1}$ | $[29,30]$ |
| $\psi$ | Rate of vaccination | 0.1 day $^{-1}$ | $[31,32]$ |
| $\alpha$ | Infection rate of humans due to consumption | 0.00 day $^{-1}$ | $[17,29]$ |
| $\beta$ | Growth rate of infective molluscs | 0.5 day $^{-1}$ | $[4,27]$ |
| $\mu$ | Mortality rate of molluscs | 1 day $^{-1}$ | $[33,34]$ |



Fig. 2. Numerical solution of system (2.1) using the parameters given in Table 1. Note the different scales on the $y$-axes.


Fig. 3. Schematic diagram of model (3.1) of clonorchiasis with human treatment and fish vaccination versus snail control.

The Jacobian matrix at $E_{0}$ is given by
$J^{I I}=\left[\begin{array}{cccccc}-\frac{\psi k_{1}+2 F_{v} r_{1}-k_{1} r_{1}}{k_{1}} & -\frac{r_{1}\left(2 F_{v}-k_{1}\right)}{k_{1}} & -\frac{r_{1}\left(2 F_{v}-k_{1}\right)}{k_{1}} & 0 & 0 & 0 \\ 0 & -\epsilon & 0 & 0 & 0 & 0 \\ \psi & 0 & -\phi & -\theta_{1} F_{v} & -\theta_{1} F_{v} & 0 \\ 0 & 0 & 0 & r_{2} & r_{2}+\omega & 0 \\ 0 & 0 & 0 & 0 & -\delta-\omega & 0 \\ 0 & 0 & 0 & 0 & \beta & -\mu\end{array}\right]$.
The eigenvalues of $J_{0}^{I I}$ are

$$
-\epsilon, r_{2},-\delta-\omega,-\mu, \pm \frac{\phi k_{1}+\psi k_{1}+2 r_{1} F_{v}-r_{1} k_{1}+\sqrt{\Gamma_{2}}}{2 k_{1}}
$$

where

$$
\begin{aligned}
\Gamma_{2}= & \phi^{2} k_{1}^{2}-2 \phi \psi k_{1}^{2}-4 \phi F_{v} k_{1} r_{1}+2 \phi k_{1}^{2} r_{1}+\psi^{2} k_{1}^{2}-4 \psi F_{v} k_{1} r_{1} \\
& +2 \psi k_{1}^{2} r_{1}+4 F_{v}^{2} r_{1}^{2}-4 F_{v} k_{1} r_{1}^{2}+k_{1}^{2} r_{1}^{2}
\end{aligned}
$$

The Jacobian matrix at $E_{1}^{I I}$ is given by
$J_{1}^{I I}=\left[\begin{array}{cccccc}a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & -\epsilon & 0 & 0 & 0 & a_{26} \\ \psi & 0 & -\phi & a_{34} & a_{35} & 0 \\ 0 & 0 & 0 & r_{2} & r_{2}+\omega & 0 \\ 0 & 0 & 0 & 0 & -\delta-\omega & 0 \\ 0 & 0 & 0, & 0 & \beta & -\mu\end{array}\right]$,
where
$a_{11}=\frac{\phi \psi-\phi r_{1}-\psi^{2}-\psi r_{1}}{\phi+\psi}, \quad a_{12}=\frac{2 \phi \psi-\phi r_{1}-\psi r_{1}}{\phi+\psi}$,
$a_{13}=\frac{2 \phi \psi-\phi r_{1}-\psi r_{1}}{\phi+\psi}, \quad a_{14}=\frac{\theta_{1}\left(\phi \psi-\phi r_{1}-\psi r_{1}\right) k_{1} \phi}{\left(\phi^{2}+2 \phi \psi+\psi^{2}\right) r_{1}}$,
$a_{15}=\frac{\theta_{1}\left(\phi \psi-\phi r_{1}-\psi r_{1}\right) k_{1} \phi}{\left(\phi^{2}+2 \phi \psi+\psi^{2}\right) r_{1}}, \quad a_{16}=\frac{\gamma\left(\phi \psi-\phi r_{1}-\psi r_{1}\right) k_{1} \phi}{\left(\phi^{2}+2 \phi \psi+\psi^{2}\right) r_{1}}$,
$a_{26}=-\frac{\gamma\left(\phi \psi-\phi r_{1}-\psi r_{1}\right) k_{1} \phi}{\left(\phi^{2}+2 \phi \psi+\psi^{2}\right) r_{1}}, \quad a_{34}=\frac{\theta_{1}\left(\phi \psi-\phi r_{1}-\psi r_{1}\right) \psi k_{1}}{\left(\phi^{2}+2 \phi \psi+\psi^{2}\right) r_{1}}$,
$a_{35}=\frac{\theta_{1}\left(\phi \psi-\phi r_{1}-\psi r_{1}\right) \psi k_{1}}{\left(\phi^{2}+2 \phi \psi+\psi^{2}\right) r_{1}}$.
The eigenvalues of $J_{1}^{I I}$ are
$-\omega,-\epsilon, r_{2},-\delta,-\mu, \frac{-\phi^{2}-\phi r_{1}-\psi^{2}-\psi r_{1}+\sqrt{\Gamma_{1}}}{2 \phi+2 \psi}$,
where

$$
\begin{aligned}
\Gamma_{1}= & \phi^{4}+4 \phi^{3} \psi-2 \phi^{3} r_{1}+10 \phi^{2} \psi^{2}-10 \phi^{2} \psi r_{1}+\phi^{2} r_{1}^{2}+4 \phi \psi^{3} \\
& -10 \phi \psi^{2} r_{1}+2 \phi \psi{r_{1}}^{2}+\psi^{4}-2 \psi^{3} r_{1}+\psi^{2} r_{1}{ }^{2}
\end{aligned}
$$

Therefore $E_{0}^{I I}$ and $E_{1}^{I I}$ is always unstable. This instability is due to the human birth rate $r_{2}>0$. The stability of $E_{2}^{I I}$ can only be shown numerically. This means that human births save the molluscs and infected fish from extinction, along with the human species itself.

Fig. 4 illustrates the time-dependent solution of system (3.1). In this case, there are damped oscillations, and the system approaches a coexistence steady state.

## 4. Model III: Logistic human growth without intervention

As a refinement of our baseline model, we assume that the susceptible humans grow logistically. Since the infection is not passed vertically, model (2.1) in this case becomes

$$
\begin{equation*}
\frac{d F_{s}}{d t}=r_{1}\left(F_{s}+F_{i}\right)\left(1-\frac{F_{s}+F_{i}}{k_{1}}\right)-\theta_{1} F_{s}\left(H_{s}+H_{i}\right)-\gamma F_{s} M \tag{4.1a}
\end{equation*}
$$

$$
\begin{align*}
\frac{d F_{i}}{d t} & =\gamma F_{s} M-\theta_{1} F_{i}\left(H_{s}+H_{i}\right)-\epsilon F_{i}  \tag{4.1b}\\
\frac{d H_{s}}{d t} & =r_{2}\left(H_{s}+H_{i}\right)\left(1-\frac{H_{s}+H_{i}}{k_{2}}\right)-\alpha \theta_{1} F_{i} H_{s}  \tag{4.1c}\\
\frac{d H_{i}}{d t} & =\alpha \theta_{1} F_{i} H_{s}-\delta H_{i}  \tag{4.1d}\\
\frac{d M}{d t} & =\beta H_{i}-\mu M \tag{4.1e}
\end{align*}
$$

System (4.1) has four biologically feasible steady states when the molluscs are eradicated:
$E_{0}^{I I I}:\left(M, F_{i}, F_{s}, H_{i}, H_{s}\right)=(0,0,0,0,0)$,
$E_{1}^{I I I}:\left(M, F_{i}, F_{s}, H_{i}, H_{s}\right)=\left(0,0, k_{1}, 0,0\right)$,
$E_{2}^{I I I}:\left(M, F_{i}, F_{s}, H_{i}, H_{s}\right)=\left(0,0,0,0, k_{2}\right)$,
$E_{3}^{I I I}:\left(M, F_{i}, F_{s}, H_{i}, H_{s}\right)=\left(0,0, \frac{k_{1}\left(r_{1}-k_{2} \theta_{1}\right)}{r_{1}}, 0, k_{2}\right)$.
Since $E_{0}^{I I I}$ and $E_{1}^{I I I}$ are always unstable, extinction of all species is not possible for positive initial conditions. It is possible that susceptible humans thrive while the remaining species become extinct if $\theta_{1}>\frac{r_{1}}{k_{2}}$ (or $R_{0}^{I I I}<1$ where $R_{0}^{I I I}=\frac{r_{1}}{\theta_{1} k_{2}}$ ). Indeed, the latter is the condition for which $E_{2}^{I I I}$ is stable. If the fish consumption or the predation rate by the human population is greater than a threshold, then all species go extinct except for susceptible humans, and the epidemic is avoided.

Theorem 4.1. The disease-free equilibrium $E_{3}^{I I I}$ is stable if and only if $r_{1}-k_{2} \theta_{1}>0$ and
$R_{0}=\frac{\gamma k_{1} k_{2} \theta \beta\left(r_{1}-k_{2} \theta_{1}\right)}{r_{1}\left(k_{2} \theta_{1}+\epsilon\right) r_{2} \mu}<1$.
Proof. The Jacobian matrix at $E_{3}^{I I I}$ is given by

$$
J_{3}^{I I I}=\left[\begin{array}{ccccc}
k_{2} \theta_{1}-r_{1} & 2 k_{2} \theta_{1}-r_{1} & \frac{k_{1} \theta_{1}\left(k_{2} \theta_{1}-r_{1}\right)}{r_{1}} & \frac{k_{1} \theta_{1}\left(k_{2} \theta_{1}-r_{1}\right)}{r_{1}} & \frac{\gamma k_{1}\left(k_{2} \theta_{1}-r_{1}\right)}{r_{1}} \\
0 & -k_{2} \theta_{1}-\epsilon & 0 & 0 & -\frac{\gamma k_{1}\left(k_{2} \theta_{1}-r_{1}\right)}{r_{1}} \\
0 & -\alpha k_{2} \theta_{1} & -r_{2} & -r_{2} & 0 \\
0 & \alpha k_{2} \theta_{1} & 0 & -\delta & 0 \\
0 & 0 & 0 & \beta & -\mu
\end{array}\right] .
$$

The two eigenvalues of $J_{3}^{I I I}$ are $k_{2} \theta_{1}-r_{1}<0,-r_{2}<0$. The remaining eigenvalues satisfy the following cubic:

$$
\begin{align*}
\rho^{3} & +\left(\delta+\mu+k_{2} \theta_{1}+\epsilon\right) \rho^{2}+\left\{\left(r_{2}+\mu\right)\left(k_{2} \theta_{1}+\epsilon\right)+r_{2} \mu\right\} \rho \\
& +r_{1}\left(k_{2} \theta_{1}+\epsilon\right) r_{2} \mu-\gamma k_{1} k_{2} \theta \beta\left(r_{1}-k_{2} \theta_{1}\right)=0 \tag{4.3}
\end{align*}
$$

Since the coefficients of Eq. (4.3) satisfy $\left(\delta+\mu+k_{2} \theta_{1}+\epsilon\right)>0$, $\left(r_{2}+\mu\right)\left(k_{2} \theta_{1}+\epsilon\right)+r_{2} \mu>0$, using Routh-Hurwitz criterion, all roots of Eq. (4.3) will be negative or have negative real parts if and only if the last coefficient is positive; i.e.,

$$
r_{1}\left(k_{2} \theta_{1}+\epsilon\right) r_{2} \mu-\gamma k_{1} k_{2} \theta \beta\left(r_{1}-k_{2} \theta_{1}\right)>0 .
$$

Fig. 5 describes the numerical simulation of system (4.1). In this case, there are essentially no oscillations, and the system approaches a coexistence equilibrium.

## 5. Model IV: Molluscicide and logistic human growth

We now build on the previous models to add molluscicide to control the snail population, while including logistic growth for humans. The model becomes

$$
\begin{align*}
\frac{d F_{s}}{d t}= & r_{1}\left(F_{s}+F_{i}+F_{v}\right)\left(1-\frac{F_{s}+F_{i}+F_{v}}{k_{1}}\right)-\theta_{1} F_{s}\left(H_{s}+H_{i}\right) \\
& -\gamma F_{s} M-\psi F_{s}  \tag{5.1a}\\
\frac{d F_{i}}{d t}= & \gamma F_{s} M-\theta_{1} F_{i}\left(H_{s}+H_{i}\right)-\epsilon F_{i} \tag{5.1b}
\end{align*}
$$



Fig. 4. Numerical solution of the system (3.1) using the parameters given in Table 1.


Fig. 5. Numerical solution of system (4.1) using the parameters given in Table 1.
$\frac{d F_{v}}{d t}=\psi F_{s}-\theta_{1} F_{v}\left(H_{s}+H_{i}\right)-\phi F_{v}$,
$\frac{d H_{s}}{d t}=r_{2}\left(H_{s}+H_{i}\right)\left(1-\frac{H_{s}+H_{i}}{k_{2}}\right)-\alpha \theta_{1} F_{i} H_{s}+\omega H_{i}$,
$\frac{d H_{i}}{d t}=\alpha \theta_{1} F_{i} H_{s}-\delta H_{i}-\omega H_{i}$,
(5.1c) $\frac{d M}{d t}=\beta H_{i}-\mu M$.
(5.1d) For the analysis of the system (5.1), we need the following invariant region:

$$
\mathcal{B}=\left\{\left(F_{s}, F_{i}, F_{v}, H_{s}, H_{i}, M\right) \in \mathcal{R}: 0 \leq F_{s}+F_{i}+F_{v} \leq k_{1},\right.
$$

$$
\begin{equation*}
\left.0 \leq H_{s}+H_{i} \leq k_{2}, H_{i} \leq \frac{\alpha \theta k_{1} k_{2}}{\delta+\omega}, 0 \leq M \leq m\right\} \tag{5.2}
\end{equation*}
$$

where
$m=\frac{\beta \alpha \theta k_{1} k_{2}}{(\delta+\omega) \mu}$.
The Jacobian matrix for the model (5.1) is given by
$J^{I V}=\left[\begin{array}{ccccc}j_{11} & r_{1}\left(1-\frac{2\left(F_{s}+F_{i}\right)}{k_{1}}\right) & -\theta_{1} F_{s} & -\theta_{1} F_{s} & -\gamma F_{s} \\ \gamma M & -\theta_{1} H_{i}-\theta_{1} H_{s}-\epsilon & -F_{i} \theta_{1} & -F_{i} \theta_{1} & \gamma F_{s} \\ 0 & -\alpha \theta_{1} H_{s} & j_{33} & r_{2}\left(1-\frac{2\left(H_{s}+H_{i}\right)}{k_{2}}\right) & 0 \\ 0 & \alpha \theta_{1} H_{s} & \alpha \theta_{1} F_{i} & -\delta & 0 \\ 0 & 0 & 0 & \beta & -\mu\end{array}\right]$,
where
$j_{11}=r_{1}\left(1-\frac{2\left(F_{s}+F_{i}\right)}{k_{1}}\right)-\theta_{1} H_{s}-\theta_{1} H_{i}-\gamma M$,
$j_{33}=r_{2}\left(1-\frac{2\left(H_{s}+H_{i}\right)}{k_{2}}\right)-\alpha \theta_{1} F_{i}$.
The Jacobian matrix at $E_{0}^{I V}$ is given by
$J_{0}^{I V}=\left[\begin{array}{ccccc}r_{1} & r_{1} & 0 & 0 & 0 \\ 0 & -\epsilon & 0 & 0 & 0 \\ 0 & 0 & r_{2} & r_{2} & 0 \\ 0 & 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & \beta & -\mu\end{array}\right]$.
The eigenvalues of $J_{0}$ are $r_{1},-\epsilon, r_{2},-\delta,-\mu$, so $E_{0}^{I V}$ is always unstable.
The Jacobian matrix at $E_{1}^{I V}$ is given by
$J_{1}^{I V}=\left[\begin{array}{ccccc}-r_{1} & -r_{1} & -k_{1} \theta_{1} & -k_{1} \theta_{1} & -\gamma k_{1} \\ 0 & -\epsilon & 0 & 0 & \gamma k_{1} \\ 0 & 0 & r_{2} & r_{2} & 0 \\ 0 & 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & \beta & -\mu\end{array}\right]$.
The eigenvalues of $J_{1}$ are $-r_{1},-\epsilon, r_{2},-\delta,-\mu$, so $E_{1}^{I V}$ is always unstable.
The Jacobian matrix at $E_{2}^{I V}$ is given by
$J_{2}^{I V}=\left[\begin{array}{ccccc}-k_{2} \theta_{1}+r_{1} & r_{1} & 0 & 0 & 0 \\ 0 & -k_{2} \theta_{1}-\epsilon & 0 & 0 & 0 \\ 0 & -\alpha k_{2} \theta_{1} & -r_{2} & -r_{2} & 0 \\ 0 & \alpha k_{2} \theta_{1} & 0 & -\delta & 0 \\ 0 & 0 & 0 & \beta & -\mu\end{array}\right]$.
The eigenvalues of $J_{2}^{I V}$ are $r_{1}-k_{2} \theta_{1},-k_{2} \theta_{1}-\epsilon,-r_{2},-\delta,-\mu$. Therefore, the system is stable if and only if $r_{1}<k_{2} \theta_{1}$.

Fig. 6 illustrates the results from system (5.1). Small oscillations appear initially before the system approaches a coexistence steady state.

## 6. Model V: Molluscicide and logistic human growth with impulsive snail control

Snail control is conducted using molluscicide in fish ponds and canals [35]. Molluscicides are most commonly applied at discrete (rather than continuous) times, in order to avoid environmental toxicity. This can be modelled as an impulsive effect $[36,37]$. Suppose that the molluscicide is applied at $t_{k}(k \in \mathbb{N})$, and the elimination rate of snails at time $t_{k}$ is $q$. With these assumptions, model (5.1) reduces to

$$
\frac{d F_{s}}{d t}=r_{1}\left(F_{s}+F_{i}+F_{v}\right)\left(1-\frac{F_{s}+F_{i}+F_{v}}{k_{1}}\right)-\theta_{1} F_{s}\left(H_{s}+H_{i}\right)
$$

$$
\begin{align*}
& -\gamma F_{s} M-\psi F_{s},  \tag{6.1a}\\
\frac{d F_{i}}{d t}= & \gamma F_{s} M-\theta_{1} F_{i}\left(H_{s}+H_{i}\right)-\epsilon F_{i}  \tag{6.1b}\\
\frac{d F_{v}}{d t}= & \psi F_{s}-\theta_{1} F_{v}\left(H_{s}+H_{i}\right)-\epsilon F_{v}  \tag{6.1c}\\
\frac{d H_{s}}{d t}= & r_{2}\left(H_{s}+H_{i}\right)\left(1-\frac{H_{s}+H_{i}}{k_{2}}\right)-\alpha \theta_{1} F_{i} H_{s}+\omega H_{i}  \tag{6.1d}\\
\frac{d H_{i}}{d t}= & \alpha \theta_{1} F_{i} H_{s}-\delta H_{i}-\omega H_{i}  \tag{6.1e}\\
\frac{d M}{d t}= & \beta H_{i}-\mu M \quad t \neq t_{k}  \tag{6.1f}\\
\Delta M= & -q M \quad t=t_{k} \tag{6.1g}
\end{align*}
$$

Taking the maximal number of infected molluscs as an upper bound, the one-dimensional impulsive differential equation takes the form:
$\frac{d M}{d t}=m-\mu M$, for $t \neq t_{k}$
$\Delta M=-q M$, for $t=t_{k}$ where $k=1,2,3, \ldots$
where $m$ is defined in (5.3).
For a single impulsive cycle $t_{k} \leq t \leq t_{k+1}$, the endpoints of Eq. (6.2) satisfy
$M\left(t_{k+1}^{-}\right)=\frac{m}{\mu}\left[1-e^{-\mu\left(t_{k+1}-t_{k}\right)}\right]+M\left(t_{k}^{+}\right) e^{-\mu\left(t_{k+1}-t_{k}\right)}$.
Here, $M\left(t_{k}^{-}\right)$and $M\left(t_{k}^{+}\right)$represent number of infected molluscs immediately before and after the impulse therapy respectively.

Solving the recurrence relation, we have

$$
\begin{align*}
M\left(t_{n}^{-}\right)= & \frac{m}{\mu}\left[(1-q)^{(n-1)} e^{-\mu\left(t_{n}-t_{1}\right)}+(1-q)^{(n-2)} e^{-\mu\left(t_{n}-t_{2}\right)}\right. \\
& +\cdots+(1-q) e^{-\mu\left(t_{n}-t_{n-1}\right)} \\
& +1-(1-q)^{(n-2)} e^{-\mu\left(t_{n}-t_{1}\right)}-(1-q)^{(n-3)} e^{-\mu\left(t_{n}-t_{2}\right)} \\
& \left.-\cdots-e^{-\mu\left(t_{n}-t_{n-1}\right)}\right] \tag{6.3}
\end{align*}
$$

and

$$
\begin{align*}
M\left(t_{n}^{+}\right)= & \frac{m}{\mu}\left[(1-q)^{n} e^{-\mu\left(t_{n}-t_{1}\right)}+(1-q)^{(n-1)} e^{-\mu\left(t_{n}-t_{2}\right)}\right. \\
& +\cdots+(1-q)^{2} e^{-\mu\left(t_{n}-t_{n-1}\right)} \\
& -(1-q)^{(n-1)} e^{-\mu\left(t_{n}-t_{1}\right)}-(1-q)^{(n-2)} e^{-\mu\left(t_{n}-t_{2}\right)} \\
& -\cdots-(1-q) e^{-\mu\left(t_{n}-t_{n-1}\right)} \\
& +(1-q)] \tag{6.4}
\end{align*}
$$

The solutions given in (6.3) and (6.4) can help to predict the infected molluscs present just before and after the $n$th impulse. However, these require the entire history of molluscicide application to be known, which is unlikely to be available.

If the time period $\tau=t_{n+1}-t_{n}$ is fixed, then we have

$$
\begin{aligned}
M\left(t_{n}^{-}\right)= & \frac{m}{\mu}\left[1+(1-q) e^{-\mu \tau}+(1-q)^{2} e^{-2 \mu \tau}+\ldots .+(1-q)^{n-1} e^{-(n-1) \mu \tau}\right. \\
& \left.-e^{-\mu \tau}\left(1+(1-q) e^{-\mu \tau}+\ldots .+(1-q)^{n-2} e^{-(n-2) \mu \tau}\right)\right] \\
= & \frac{m}{\mu}\left[\frac{1-(1-q)^{n} e^{-n \mu \tau}}{1-(1-q) e^{-\mu \tau}}-e^{-\mu \tau} \frac{1-(1-q)^{n-1} e^{-(n-1) \mu \tau}}{1-(1-q) e^{-\mu \tau}}\right] .
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\lim _{n \rightarrow \infty} M\left(t_{n}^{-}\right) & =\frac{m}{\mu}\left[\frac{1}{1-(1-q) e^{-\mu \tau}}-e^{-\mu \tau} \frac{1}{1-(1-q) e^{-\mu \tau}}\right] \\
& =\frac{m}{\mu}\left[\frac{1-e^{-\mu \tau}}{1-(1-q) e^{-\mu \tau}}\right]
\end{aligned}
$$

This is the maximum number of infected molluscs in the long term before applying the molluscicide.

After applying the molluscicide, the number of the infected molluscs is expressed by

$$
\lim _{n \rightarrow \infty} M\left(t_{n}^{+}\right)=(1-q) \lim _{n \rightarrow \infty} M\left(t_{n}^{-}\right)
$$



Fig. 6. Numerical solution of the system (5.1) using the parameters given in Table 1.

$$
=(1-q) \frac{m}{\mu}\left[\frac{1-e^{-\mu \tau}}{1-(1-q) e^{-\mu \tau}}\right]
$$

This is the long-term minimum number of infected molluscs.
Suppose $\tilde{M}$ is the maximum acceptable level of infected molluscs. After long-term molluscicide application, in order to keep the infected molluscs below $\tilde{M}$, we need the maximum time interval, $\tau_{\max }$, between two consecutive impulses to satisfy
$(1-q) \frac{m}{\mu}\left[\frac{1-e^{-\mu \tau}}{1-(1-q) e^{-\mu \tau}}\right]<\tilde{M}$.
Hence
$\tau<\frac{1}{\mu} \ln \left[\frac{\tilde{M} \mu-(1-q) m}{(1-q)(m-\tilde{M} \mu)}\right]=\tau_{\max }$.
From (6.5), we have the maximum time interval of molluscicide application that will keep the infected molluscs below any arbitrary level $\tilde{M}$. However, we cannot clear the molluscs entirely without spraying infinitely often, which is not feasible (and the impulsive assumptions do not apply, in any case).

Fig. 7 illustrates system (6.1), showing the effect of applying impulsive control (in red). The impulsive molluscicides quickly control the infection if applied frequently enough.

## 7. Model behaviour comparison

Comparing the coexistence equilibria of model (2.1) with model (4.1), the susceptible human population is bounded. This is notable for model (2.1), where the human growth rate is assumed to be of Malthus type. However, the presence of the other populations keeps the possible explosion of the humans in check.

The disease is endemic in both situations, with all populations thriving. However, the presence of a carrying capacity $k_{2}$ for humans induces a slightly higher equilibrium level, mainly in the infected human subpopulation, and consequently also in their cumulative numbers. The human carrying capacity entails a higher prevalence for the logistic case; it is roughly $3 /(3+4)=0.43$, while for the Malthus case, it reduces to $1 /(3+1)=0.25$. The mollusc population settles at a higher
level in the logistic case, from 10 to 30 . The susceptible fish population is significantly reduced, from 700 in the Malthusian formulation to about 500 for the human logistic case. The prevalence in the two cases is respectively $200 /(700+200)=0.22$ and $350 /(500+350)=0.41$. Thus the introduction of the human logistic behaviour actually worsens the epidemic situation for the fish.

Comparing instead (3.1) to (2.1), the susceptible human population is also bounded, and the disease is endemic. However, the presence of the fish vaccination induces a higher equilibrium level for the humans, in both subpopulations of healthy and infected, and consequently also in their cumulative numbers. However, with the human carrying capacity, the disease attains a higher prevalence; it is roughly $3 /(3+7)=0.3$ in the latter case and $1 /(3+1)=0.25$ in the former. The mollusc population is higher in the vaccination case, raising to a value around 17 from 10. The fish population benefits the most from the introduction of the vaccination policy, which is not unexpected. It jumps from about 900 to 7000, with the prevalence exhibiting a moderate decrease, from $200 / 900=0.22$ to $1000 / 7000=0.14$.

We now compare models (4.1) and (5.1). Both contain the human population dynamics formulated via a logistic model, but a molluscicide is used in the second one. Molluscs are reduced from 30 to a value around 15. For the human population, the changes are very small for both susceptible and infected individuals, entailing also little change in the disease prevalence. For the healthy fish, there is a sixfold raise if vaccination is applied, from 500 to 3000 . Their overall population also experiences a large increase, from 900 to 7000 . The fish disease prevalence drops dramatically from $400 / 900=0.44$ to $1000 / 7000=$ 0.14 .

We now consider models (3.1) and (5.1) where vaccination is used in both, but the former has a Malthusian formulation for the humans, and the latter has a logistic formulation. The mollusc population does not show any significant change. The humans are at level 10 in the former case, and about the same in the latter. This is true for both healthy and infected subpopulations, which implies that the disease prevalence does not change, remaining at level $3 /(3+7)=0.3$. Infected

 the system with impulsive control. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
fish are 1000 in the former case, with a prevalence of $1000 / 7000=0.14$, and the situation does not seem to change much in the second case.

Finally, we take into consideration the model with impulsive snail control, system (6.1). This policy allows us to keep all infected populations in check. Essentially, the molluscs are controlled (albeit not eradicated), and the infection is eliminated in humans. The enlargements of Fig. 7 show that there is indeed a small epidemic outbreak in both humans and fish before the disease gets eradicated. However, in comparison with the model without impulses (5.1), the peaks are reduced by a factor of 40 for the fish and somewhat reduced for the other two populations. In all the populations, the disease is extinguished quickly, in a smooth decreasing fashion.

## 8. Discussion

Clonorchiasis has been recognized as a neglected tropical disease by the World Health Organization for decades and remains prevalent worldwide. Clinical and epidemiological research into clonorchiasis over the past 140 years has contributed to a deeper understanding of the parasite, intermediate hosts and disease $[38,39]$. Most of the clonorchiasis studies focus on the discovery of new diagnostic, drug and vaccine targets, as well as the pathology and the biology of the disease. Relatively few studies have used mathematical models to assess strategies for large-scale control of clonorchiasis.

We studied the dynamics of clonorchiasis using four deterministic mathematical models to describe the human-snail-fish transmission of clonorchiasis. Our models examined the current control and prevention policies and incorporated impulsive snail control strategy for reducing the transmission of clonorchiasis. We also derived the reproduction number for the general epidemic model. Using the theory of impulsive differential equations, we found an extinction threshold for the disease, along with the maximum interval of applying molluscicides. We also
performed numerical simulations to examine multiple scenarios and found that fish vaccination and snail control are both viable strategies. Other possible intervention methods could include sanitary deconfinement [40] or mimicking the control used for COVID-19 [41]. A potential modelling direction is to develop a digital twin for clonorchiasis, linking timescales of system time and slow time [42].

Due to the neglect and absence of systematic interventions, clonorchiasis remains prevalent worldwide, although some chemotherapy and control programmes have been implemented over several years in a few endemic areas $[18,43]$. Further research is needed to increase the number of models and to address the neglect of this disease in the field.

## CRediT authorship contribution statement

Mainul Haque: Co-designed the study. Fahad Al Basir: Mathematical analysis, Numerical simulations. Ezio Venturino: Edited the manuscript. Abdulhalim Saeed: Mathematical analysis, Numerical simulations. Stacey R. Smith?: Co-designed the study, Wrote the introduction, Edited the manuscript.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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