MAT4996: Assignment 1. Due January 25th.

1. Zombies have invaded campus! Initially, there are 5 zombies. They recruit more of the undead to their ghoulish ranks at rate

$$\frac{dz}{dt} = 10te^{-0.08t},$$

where t is the time in days and z are the number of zombies.

   (a) How many zombies are recruited in the first week?

   (b) How many zombies are recruited during the third week?

   (c) After 50 days, how many zombies are there in total?

   (d) Will zombies eventually infect everyone on campus? If not, how many will be infected eventually?

2. Simple zombie conversion that does not cause death may be modelled by the pair of differential equations

$$\begin{aligned} S' &= -\beta SZ \\ Z' &= \beta SZ \end{aligned}$$

Show that, in a population of fixed size K, everyone will become a zombie.

3. Suppose zombies infect some humans at rate β, but kill others at rate λ (and do not resurrect them). If humans permanently kill zombies at rate α, then the system is modelled by

$$\begin{aligned} S' &= -\beta SZ - \lambda S \\ Z' &= \beta SZ - \alpha Z\,. \end{aligned}$$

Show that both S and Z approach zero as $t \to \infty$. (Hint: you need to think about global stability, not just local stability.)

4. Suppose we've invented a cure for zombieism. Initially, there are Z_0 zombies and a total population of N individuals. We kill zombies at a rate r that is proportional to the total zombie population. The rate at which the disease spreads, β, is equal to 6.90675×10^{-5}. The kill rate of zombies is 0.1 per day and $N = 10,000$ people. Initially, there are 20 zombies. The cure can immunize zombies so that they cannot be infected again and is applied at a constant rate of $b = 10$ zombies per day. You can assume that the timescale is short so that human births and deaths can be ignored.

   (a) Draw the flow diagram for this system. Justify any choices you make.

   (b) Write down the differential equations (using parameters, not numbers).

   (c) What are the units of β?

   (d) Find the disease-free equilibrium.

   (e) Does the system have an endemic equilibrium? If so, find it. If not, prove that one does not exist.

   (f) For the specific parameter values, what will happen in the long term?

   (g) Discuss any limitations on the assumptions in this model.