The R_0 sleight of hand

There's nothing actually incorrect, but there are a couple of dodgy bits. Moving "the negatives" to one side isn't as obvious as it might seem, since there's nothing inherent about positive or negative values. For example, we could apply the same reasoning if we simply added and subtracted

$$\begin{aligned} aN + 5 - 5 - b &< 0 \\ aN + 5 &< 5 + b \\ \frac{aN+5}{5+b} &< 1 \end{aligned}$$

and then we could define an "R_0^{SIS2}" to be $\frac{aN+5}{5+b}$. This would have the same threshold properties (ie if $R_0^{\text{SIS2}} < 1$ then the disease dies out, whereas if $R_0^{\text{SIS2}} > 1$ then the disease will become endemic), but it clearly isn't the same value. Furthermore, it is highly unlikely to be the average number of secondary infections (since adding and subtracting 5 was pretty arbitrary).

We could obviously define an infinite number of threshold parameters in this way. However, there's more to it than that. When we have the condition

$$\frac{aN}{b} < 1\,,$$

it's by no means clear that we must necessary define R_0^{SIS} the way we did. For instance, we could just as easily define

$$R_0^{\text{SIS3}} = \left[\frac{aN}{b}\right]^2$$

and we'd *still* have a threshold parameter with the right properties that is again unlikely to be the average number of secondary infections (since we arbitrarily squared).

In fact, how do we even know that our original value R_0^{SIS} is the average number of secondary infections? Answer: we don't. The whole question of matching the R_0 values derived from ODE models to the 'true' R_0 is a fascinating and lengthy one and we've only just scratched the surface.