# Partial fractions

In order to solve

$$\frac{dI}{(A - aI)I} \quad = \quad dt\,,$$

we need to split the fraction $\frac{1}{(A-aI)I}$ into simpler fractions (because we don't know how to integrate the left hand side as it stands).

These fractions will be some combination of $\frac{1}{A-aI}$ and $\frac{1}{I}$. We don't yet know what kind of combination, so let's label the unknowns by $G$ and $H$:

$$\frac{G}{A - aI} + \frac{H}{I} \quad = \quad \frac{1}{(A - aI)I}\,.$$

We're trying to solve for $G$ and $H$ so let's multiply everything by the common denominator (which is $(A - aI)(I)$). This means we get

$$IG + (A - aI)H \quad = \quad 1\,.$$

Because $G$ and $H$ are constants, this equation must hold true no matter what values of $I$ we pick. So let's be clever and pick $I = 0$ (since that will eliminate the $G$ part) and $I = \frac{A}{a}$ (since that will eliminate the $H$ part). Thus

$$\begin{array}{rrcll} I = 0: & 0 + AH & = & 1 & \Longrightarrow H = \dfrac{1}{A} \\ I = \dfrac{A}{a}: & \dfrac{A}{a}G + 0 & = & 1 & \Longrightarrow G = \dfrac{a}{A}\,. \end{array}$$

Substituting our values for $G$ and $H$ into the original fractions, we have

$$\frac{1}{(A - aI)I} \quad = \quad \frac{a}{A(A - aI)} + \frac{1}{AI}$$

This is much, much easier, because we know how to integrate the right hand side.