

# MAT4996 Assignment 3

1. Consider the SIRS model

$$\begin{aligned}S' &= -\beta SI + \nu R \\I' &= \beta SI - \gamma I \\R' &= \gamma I - \nu R\end{aligned}$$

- (a) Name a disease that this model might describe.
- (b) Find all equilibria of the system.
- (c) Find  $R_0$
- (d) Determine the stability of each equilibrium for  $R_0 < 1$  and  $R_0 > 1$ .

2. Consider the SIS model with disease-specific death

$$\begin{aligned}S' &= -\beta SI + \gamma I \\I' &= \beta SI - (\gamma + \alpha)I\end{aligned}$$

- (a) Name a disease that this model might describe.
- (b) Show that

$$R_0 = \frac{\beta N_0}{\gamma + \alpha},$$

where  $N_0$  is the population at the beginning of the outbreak.

- (c) Show that  $S' < 0$  when  $S = \frac{\gamma + \alpha}{\beta}$  and  $I \neq 0$ .
- (d) (BONUS) Deduce that

$$\lim_{t \rightarrow \infty} I(t) = 0.$$

(Hint: sketch the  $S$ - $I$  phase plane and find the nullclines. You also need to rule out the existence of any periodic orbits.)

- (e) Is this a contradiction if  $R_0 > 1$ ? Explain.

3. Consider the model

$$\begin{aligned}S' &= b(a - S) - \beta SI + \gamma I \\I' &= \beta SI - \gamma I - bI\end{aligned}$$

- (a) Describe in words the biological behaviour of this model.
- (b) Under what conditions will the disease become endemic? (Express your answer in words a nonmathematician would be able to understand.)
- (c) Is the total population size affected by disease?