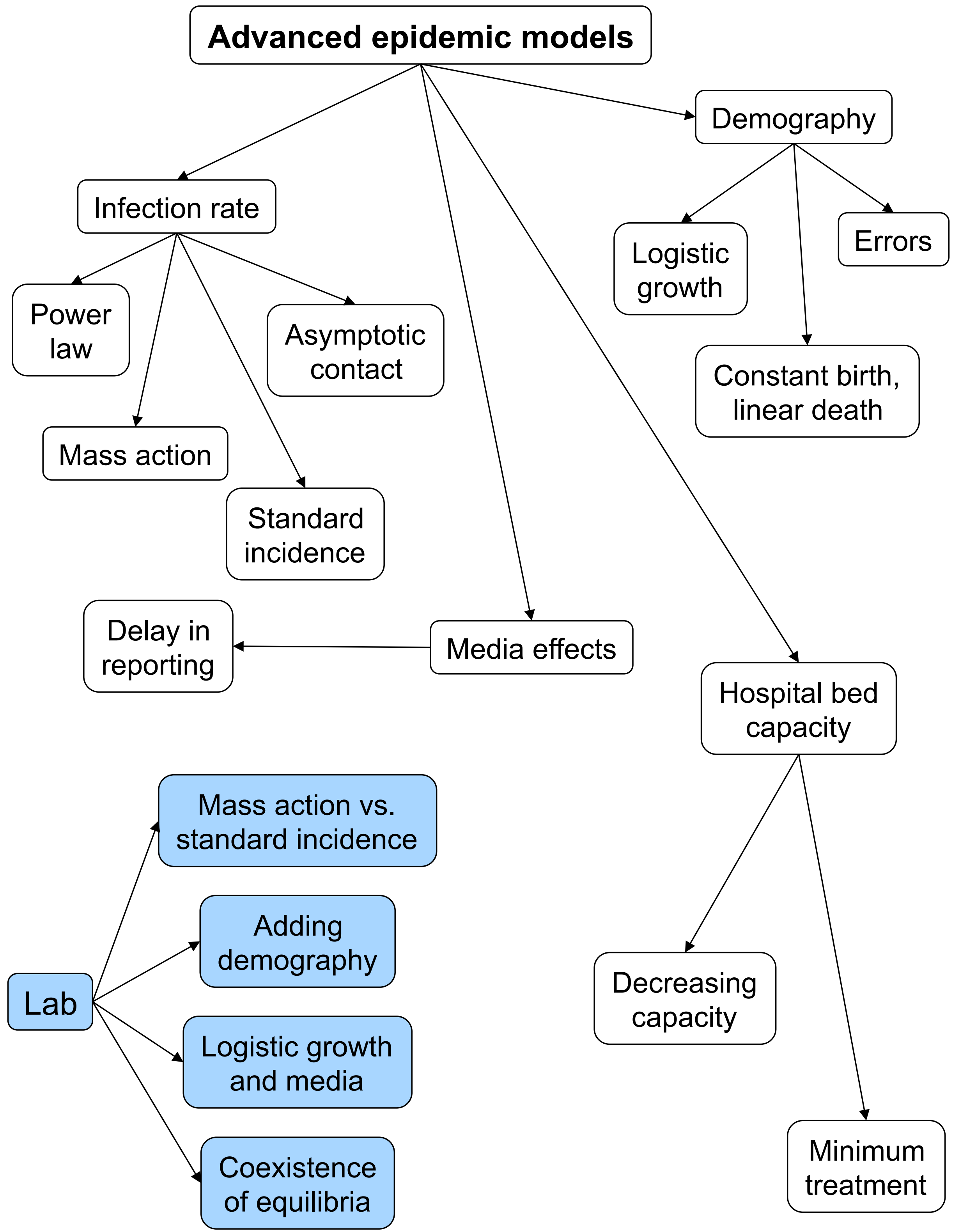
Advanced epidemic models

- Forms of the infection rate
- Adding demography
- The effects of media
- Hospital bed capacity.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Infection rate
Demography
Power law
Mass action
Asymptotic contact
Standard incidence
Logistic growth
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Hospital bed capacity
Lab
Mass action vs. standard incidence
Adding demography
Logistic growth and media
Coexistence of equilibria
Decreasing capacity
Minimum treatment

Mass-action transmission

- Form of infection: $S' = -\beta SI$
- No transmission if S=0 or I=0
- But this isn't the only possible formulation
- Assumes a well-mixed population
- E.g., influenza
- It's density dependent
- The more people infected, the more likely you are to catch it
- The contact rate thus depends on the density of people around you.

S=Susceptible,
I=Infected,
β=transmission

Standard incidence

- Form of infection: $S' = -cSI/(S+I)$
- E.g., HIV
- It's frequency dependent
- It doesn't matter how many people have the disease around you, only how many you come into contact with
- Unlike mass action, it's bounded if the population gets large
- If $N=S+I$ is constant, then this is equivalent to mass action.

S=Susceptible,
I=Infected,
c=contact rate

Power relationship

- Form of infection: $S' = -\beta S^p I^q$
  - where p and q are two further parameters we might have some control over
- Another choice that satisfies the condition that no transmission occurs if S=0 or I=0
- E.g., if it takes two zombies to infect a single human, then p=1 and q=2.

S=Susceptible, I=Infected, β=transmission

Asymptotic contact

- Form of infection: $S' = -\frac{\beta S^p I^q}{1 - \epsilon + \epsilon(S + I)}$
  where ϵ ranges between 0 and 1
- If ϵ = 0, we have the power relationship
- If ϵ = 1, we have a generalised form of standard incidence
- Note that S and I must be proportions (and hence unitless)
- This generalises all previous models
- But the possibilities are endless.

S=Susceptible, I=Infected, β=transmission

Model choice

- The choice of model depends on the biology
- There's also the issue of mathematical tractability
  - we need to be able to analyse these models
- Or we may perform a purely numerical analysis, though that can miss things
- Even simple models can give rise to very complicated behaviour
- Sometimes we may trade accuracy for insight.

Demography

- We want to include births and deaths
  - immigration can be subsumed into “births”
- In the absence of infection, we could write $S'=\pi(S)-dS$

  where $\pi(S)$ is a growth function and d is the background death rate
- It turns out that this means individuals are alive for 1/d time units
- This is the easiest way to include the death rate.

S=Susceptible

Constant death?

- Why not $S' = \pi(S) - d$?
- Answer: because populations could becme negative
- E.g., suppose $S(0)=0$ and $\pi(0)=0$ (if there's nobody around, the population can't grow)
- Then $S'(0)=-d<0$
- Hence the population will decrease from 0, becoming negative
- We'd like to avoid negative people ☺.

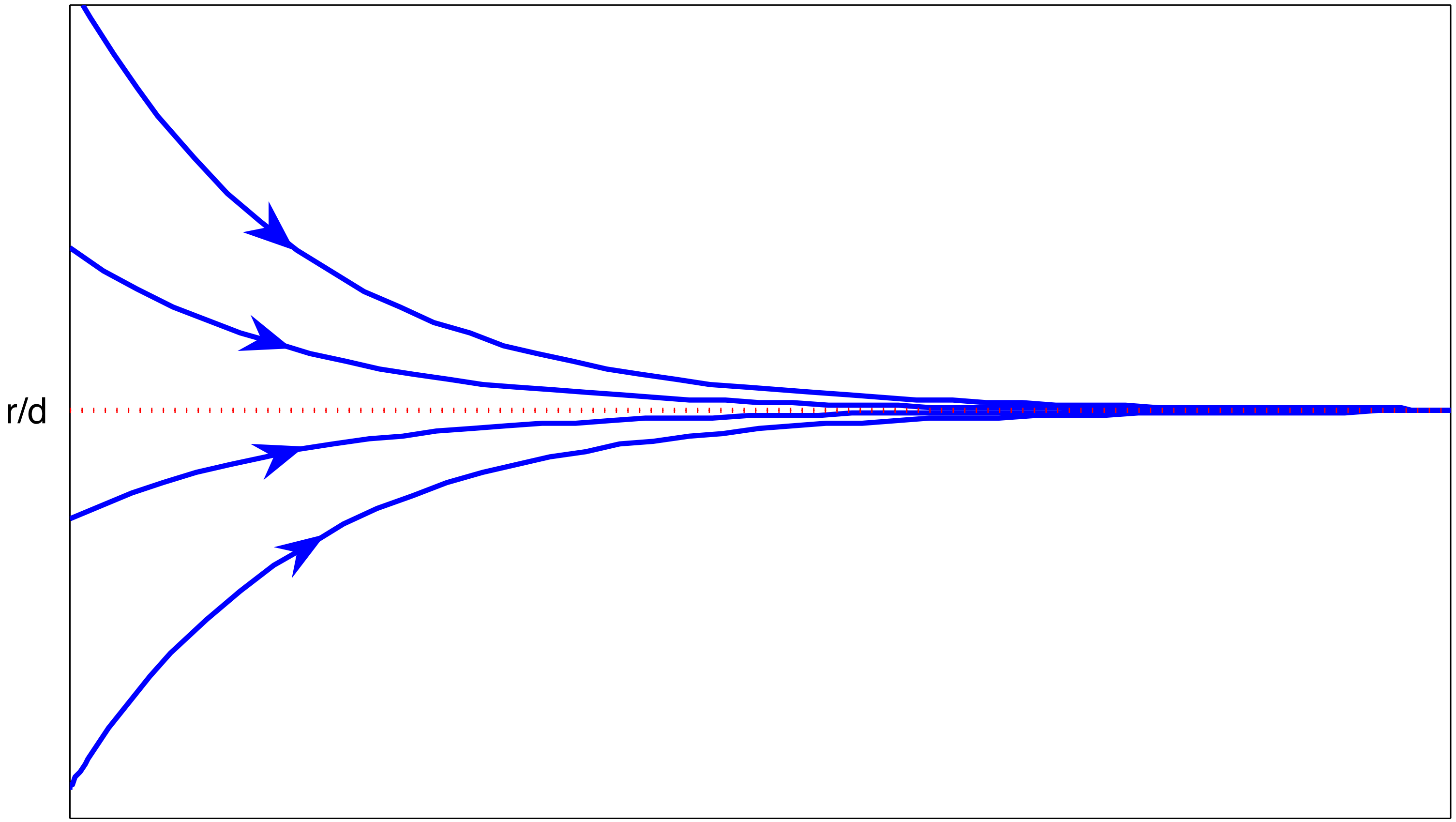
S=Susceptible,
π=growth,
d=death

Constant birth, linear death

- The birth rate can be constant
- Hence $S'=r-dS$
- Equilibrium at $S=r/d$
- $S'<0$ if $S>r/d$ while $S'>0$ if $S<r/d$
- For this simple model, all solutions approach the equilibrium.

S=Susceptible, r=growth, d=death

All solutions converge



- All solutions tend towards r/d monotonically.

r=growth, d=death

Logistic growth

- In this case, the growth rate is given by

$$\pi(S) = rS\left(1 - \frac{S}{K}\right)$$

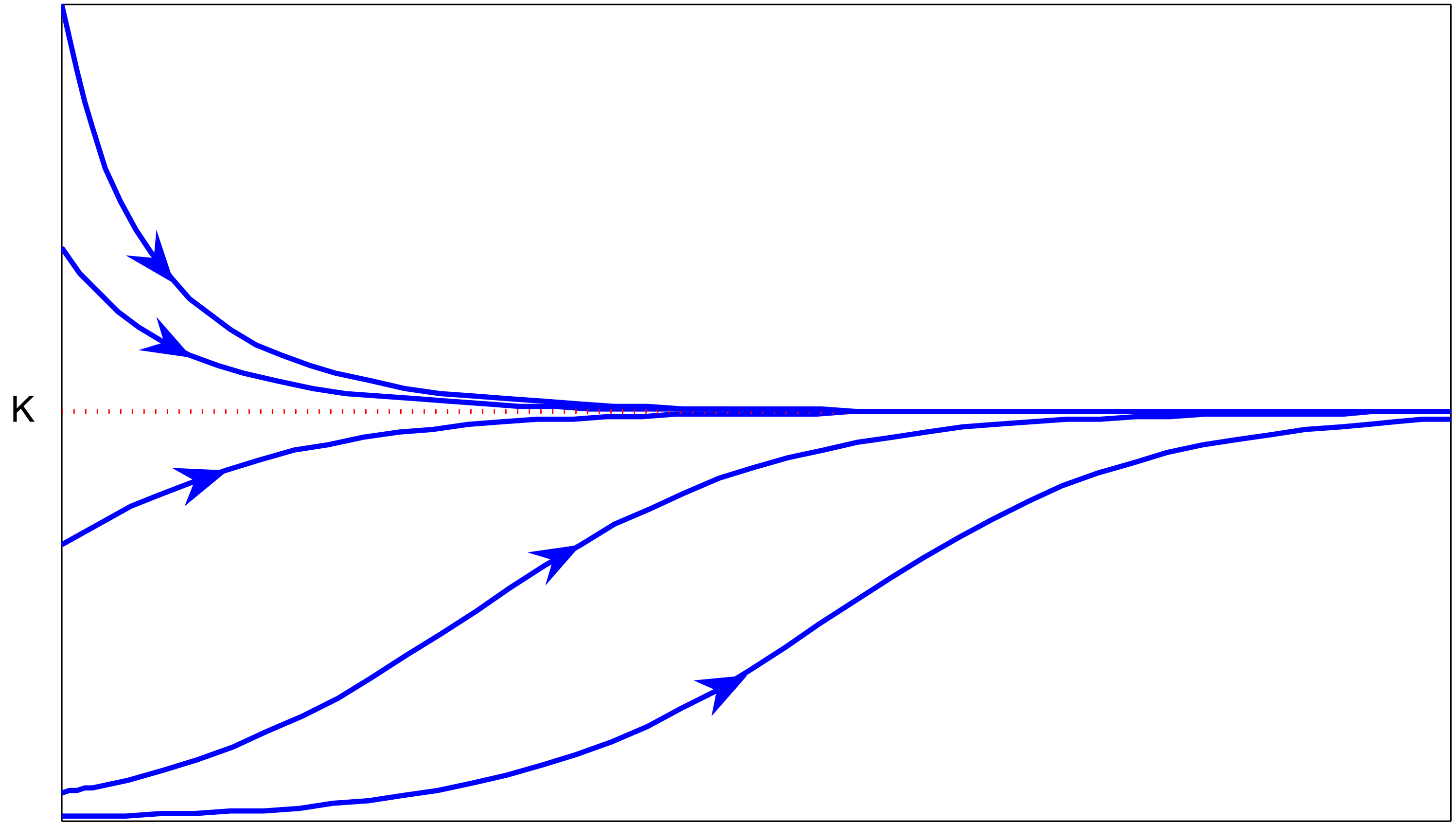
- Since there's already a linear factor rS, we can absorb the -dS term into this
- Hence we have

$$S' = rS\left(1 - \frac{S}{K}\right)$$

- There is no change if S=0 or S=K
- If $S<K$, then $S'>0$, and if $S>K$, then $S'<0$
- We call K the carrying capacity.

S=Susceptible, r=growth

Logistic growth



- Solutions that start with small initial conditions are "flatter" at first
- There's a point of inflection that changes some solutions from concave up to concave down.

K=carrying capacity

Linear growth, linear death

- Previously we had constant birth and linear death
- The logistic term had nonlinear growth and linear death
- Why not linear growth and linear death?
- E.g., $S' = rS - dS$
- However, it turns out that there are problems when both terms are of the same order.

S=Susceptible, r=growth, d=death

Explicitly solving

- Rewriting, we have $S' = (r-d)S$
- The solution is $S(t) = S(0)e^{(r-d)t}$
- If $r > d$, solutions increase to infinity (impossible)
- If $r < d$, the entire population dies out (unlikely)
- What if $r = d$? Would that take care of it?
- Unfortunately, this is a knife-edge case
- Tiny fluctuations in the birth or death rate can result in catastrophic consequences.

S=Susceptible, r=growth, d=death

The effects of media

- The media is a powerful tool for affecting people's behaviour
- The media can go into overdrive when there's a pandemic
- Or they might ignore the disease for longer than they should
- The response is not always straightforward, however.

Including behaviour changes

- We’ll focus on the transmission rate, although it’s not the only possibility
- It might change due to people mixing less as a result of media reporting on the disease
- The more people infected, the lower the contact rate and hence the transmission rate
- In a pandemic, you still have to go to work, but you might not go to a hockey game.

The model

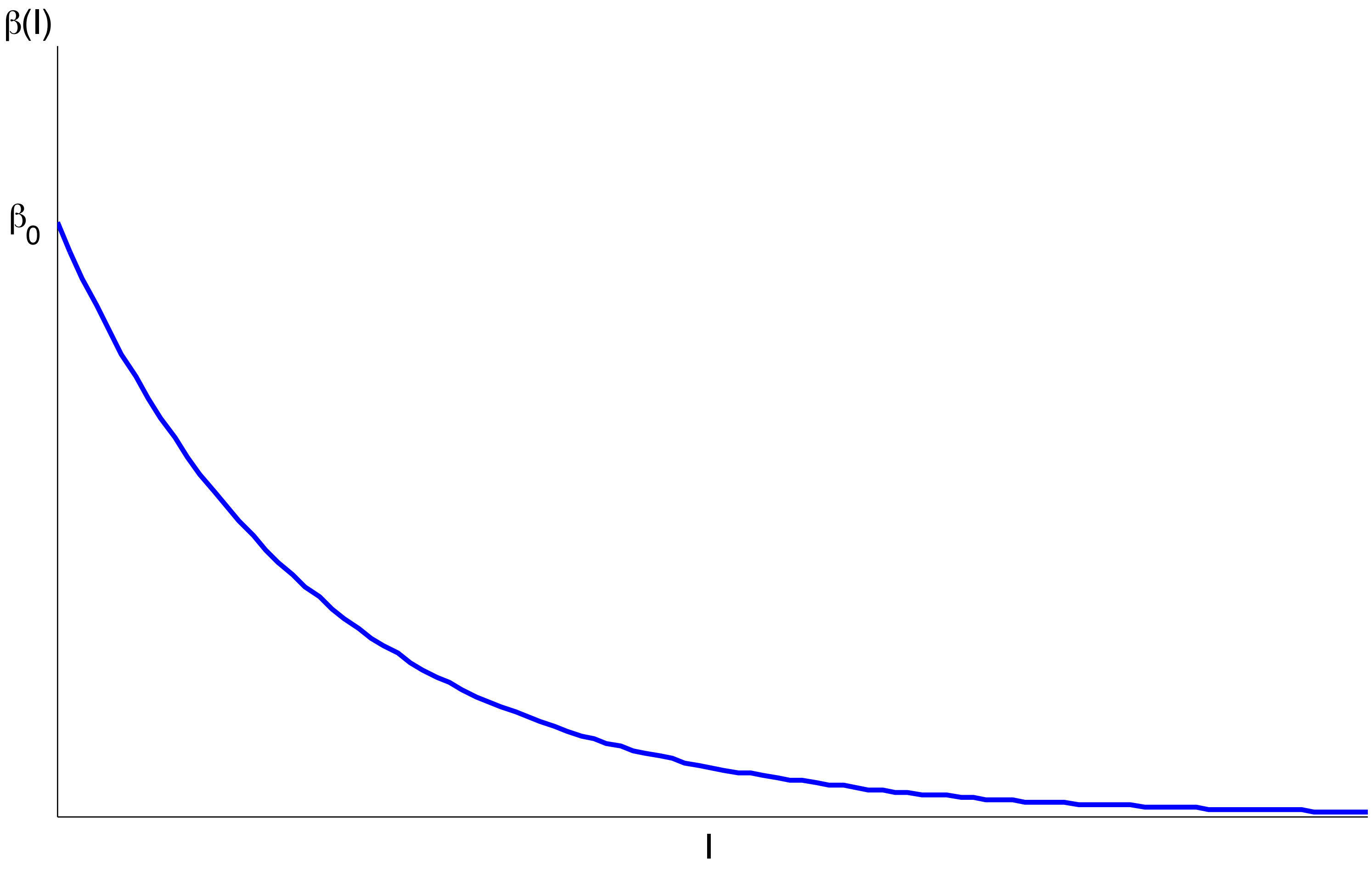
- Using (say) logistic growth and standard incidence, our model might look like

$$S' = rS\left(1 - \frac{S}{K}\right) - \beta(I)\frac{SI}{S+I}$$

  where β is no longer a constant
- For example, $\beta(I) = \beta_0 e^{-mI}$ so that the transmission function decreases as the number of infected individuals increases
- It eventually approaches zero as the whole population becomes infected.

S=Susceptible,r=growth, K=carrying capacity, β=transmission

Transmission decreases uniformly



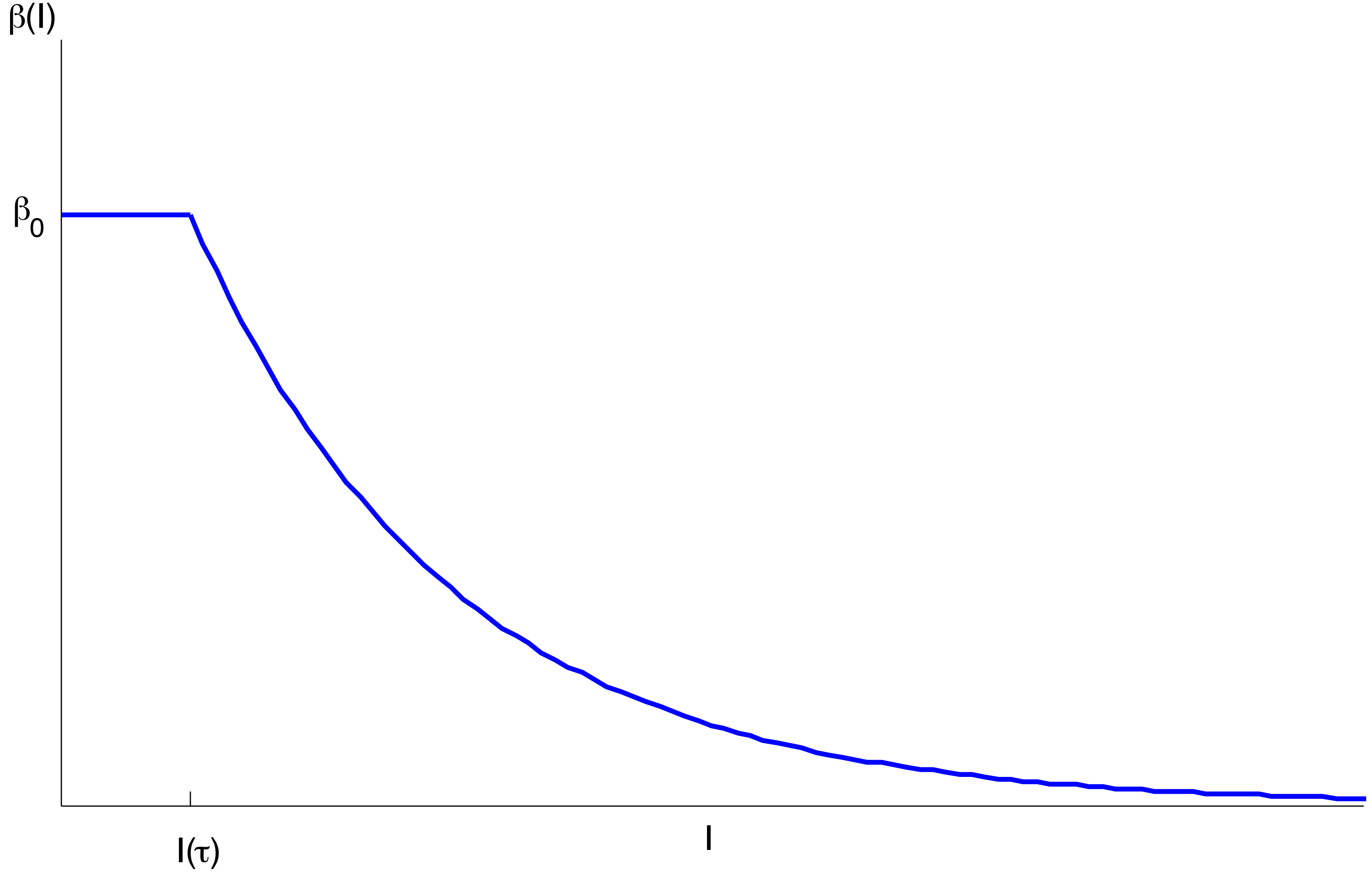
β=transmission,
I=infecteds

Delays in health reporting

- The media response is not instantaneous
- Information takes time to be released and reported
  - even in the 24 hour news cycle ☺
- If it takes τ days for the health system to release numbers of infected people, then that will introduce a delay into the system
- The media transmission function is then $\beta(I) = \beta_0 e^{-mI(t-\tau)}$
  - (I depends on time, so β does as well).

β=transmission,
m=media decay
I=infecteds, τ=delay

Transmission decreases after a delay



β=transmission,
I=infecteds, **τ**=*delay*

Hospital bed capacity

- As well as the infection rate and demography, we might have control over the recovery rate
- Medical resources will determine the treatment and recovery rate
  - but resources may be limited
- We'll use the hospital bed capacity as a proxy
- It has a maximum and a minimum
  - eg hospitals may get inundated in a pandemic, but some mimimum treatment will still occur.

The model

- We'll consider an SIR model

$$S' = r - dS - \frac{\beta SI}{S+I+R}$$

$$I' = \frac{\beta SI}{S+I+R} - dI - \nu I - \mu(b,I)I$$

$$R' = \mu(b,I)I - dR$$

S=Susceptible,I=infecteds, R=recovered, r=growth, d=death, β=transmission, b=beds

- Our standard incidence denominator involves all populations
- The disease-specific death rate is ν
- The recovery rate μ depends on the hospital beds and the number of infected individuals.

Conditions on μ(b,I)

We want the following conditions:

- μ is positive for b > 0
- $\mu(b,0) = \mu_1 > 0$, the maximum per captia recovery rate due to sufficient resources
- As I increases, μ decreases
- Some minimum number of individuals will get treated even during a pandemic, so

$$\lim_{I \to \infty} \mu(b, I) = \mu_0 > 0$$

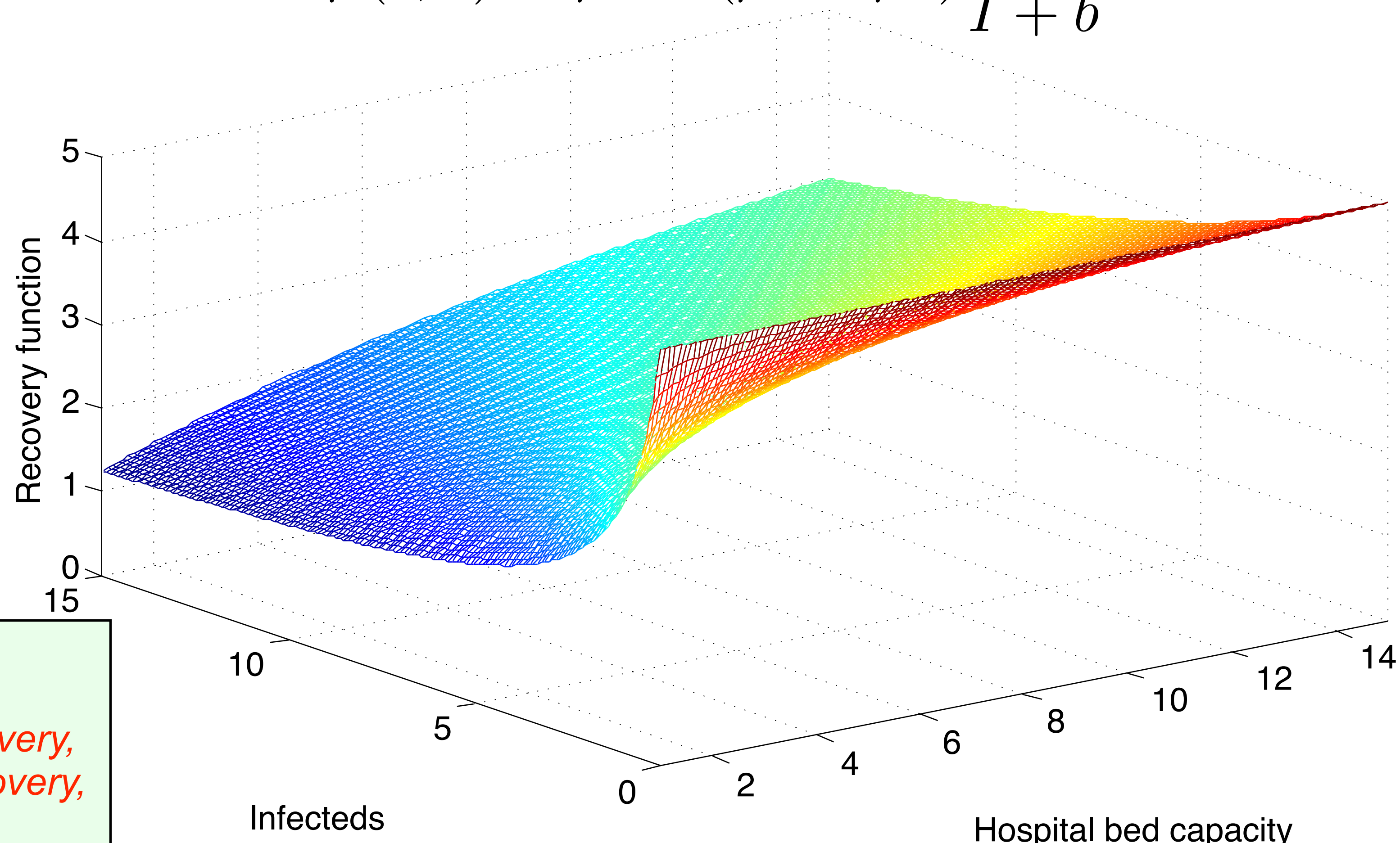
- As b increases, μ increases
  - more beds = more recovery.

I=infecteds, μ=recovery, b=beds

The recovery function

- The simplest function that satisfies these requirements is

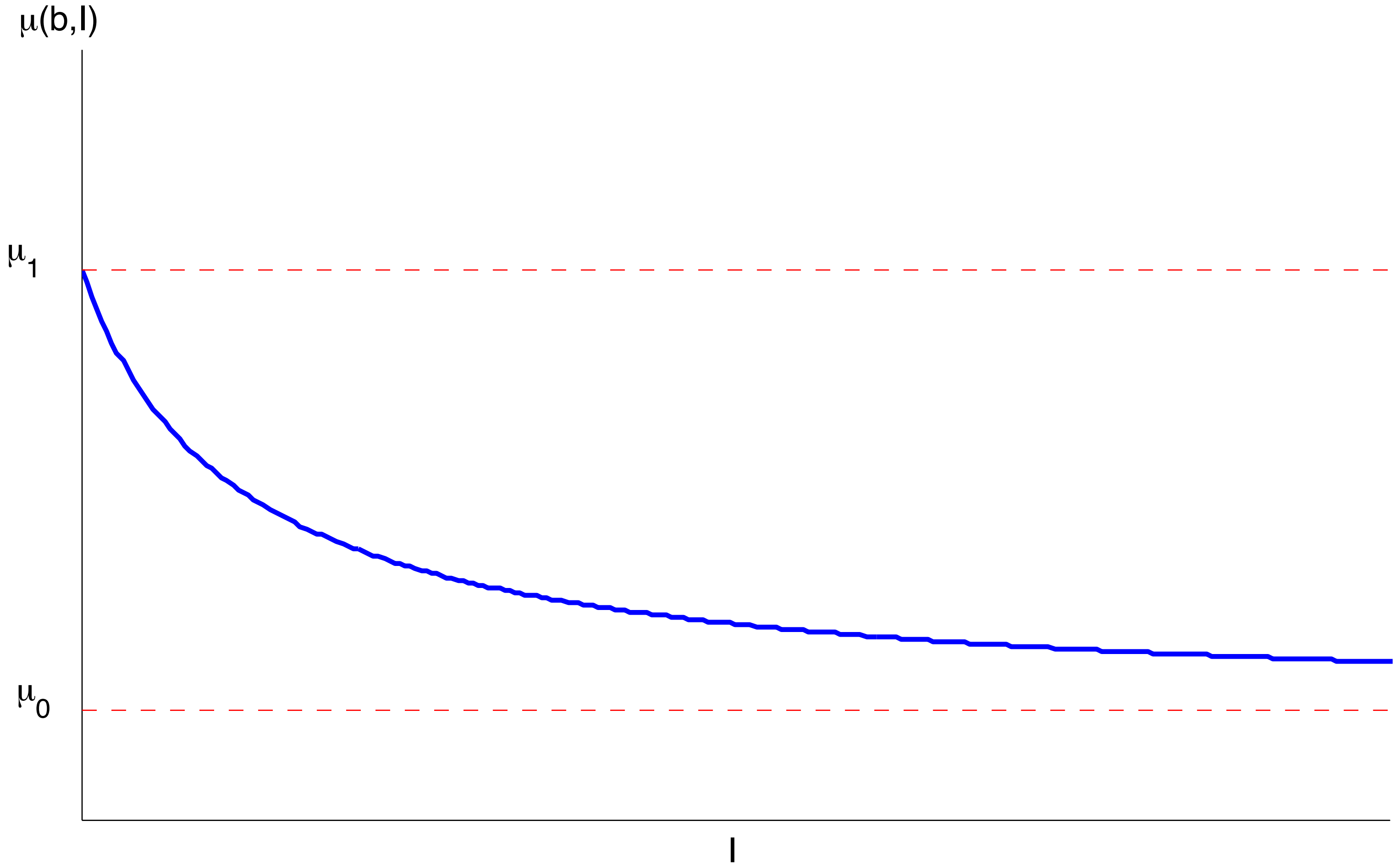
$$\mu(b, I) = \mu_0 + (\mu_1 - \mu_0)\frac{b}{I + b}$$



I=infecteds,
μ=recovery,
μ_0=min recovery,
μ_1=max recovery,
b=beds

Recovery as a function of I

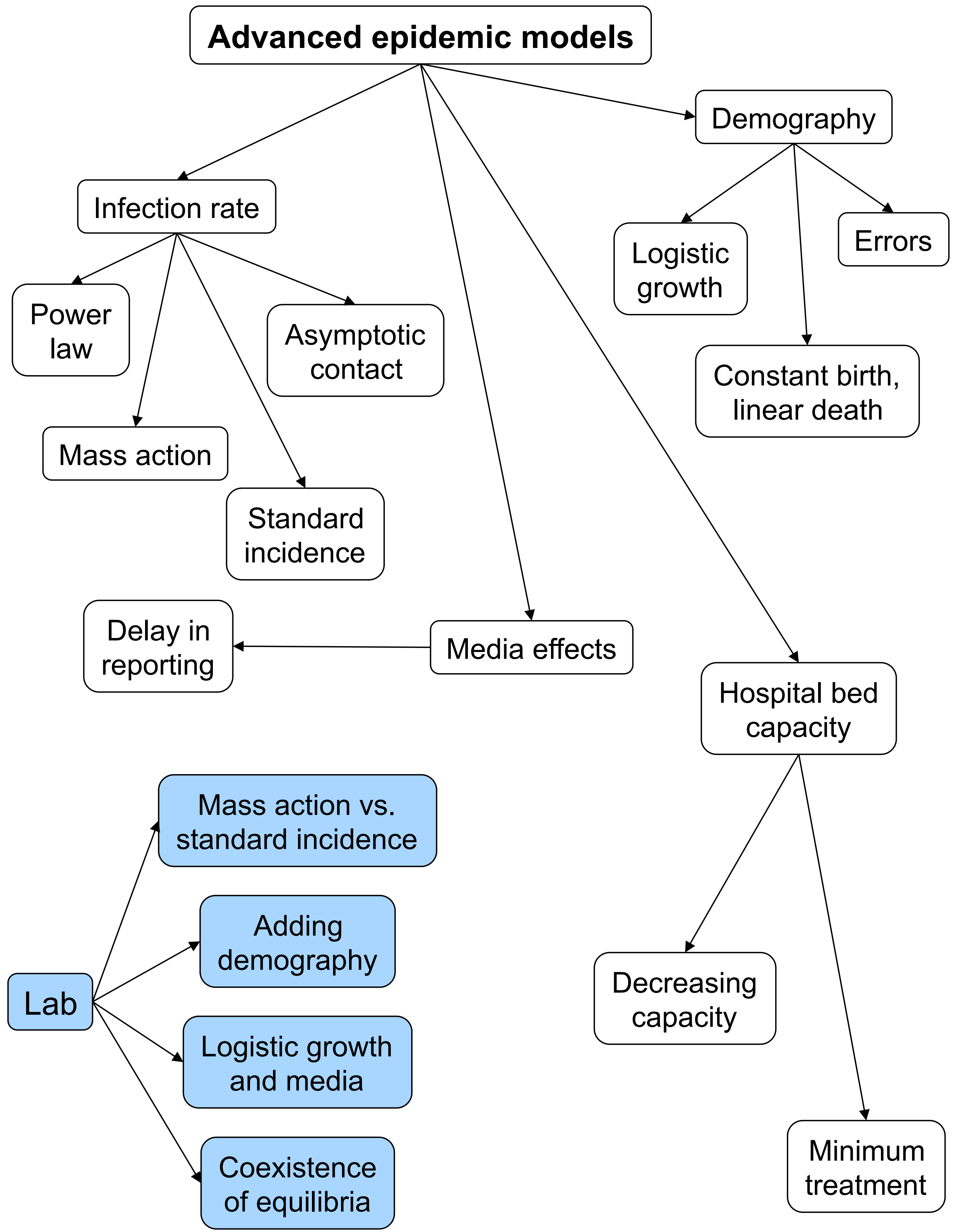
- For a given number of hospital beds, the recovery function looks like this:



μ=recovery,
μ_0=min recovery,
μ_1=max recovery,
b=beds

Lab work

- In the lab, we'll determine some of the differences between mass action and standard incidence
- We'll add demography and look at the effects of media
- We'll also analyse the basics of the hospital bed model.



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