McKay's correspondence and characters of finite subgroups of SU(2)

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In Honor of Jacques Carmona

ABSTRACT According to MacKay [1980] the irreducible characters of finite subgroups of SU(2) are in a natural 1-1 correspondence with the extended Coxeter-Dynkin graphs of type ADE. We show that the character values themselves can be given by a uniform formula, as special values of polynomials which arise naturally as numerators of Poincaré series associated to finite subgroups of SU(2) acting on polynomials in two variables. These polynomials have been the subject of a number of investigations, but their interpretation as characters has apparently not been noticed.

1 Introduction

In 1980 McKay announced his astounding discovery that the finite subgroups of SU(2) are in natural 1-1 correspondence with the extended Coxeter-Dynkin graphs of type ADE in the following way. Let K be a finite subgroup of SU(2), $\{\chi_i\}$ its irreducible characters, and χ the character of its natural representation on $\mathbb{C}^2$. Let $M = (m_{ij})$ be the matrix defined by

$$\chi\chi_i = \sum_j m_{ij}\chi_j.$$

The matrices M corresponding to the finite subgroups of $SU(2)$ exactly the matrices of the form $M = 2I - C$ where C is the Cartan matrix of an extended Coxeter-Dynkin graph of type ADE. McKay apparently found and verified this fact by direct computation. In the meantime there have been many attempts to explain it in other ways or to provide further insight into this phenomenon. Steinberg offered an explanation in terms of representation theory of finite groups in 1982. Gonzales-Sprinberg and J.-L. Verdier [1983] gave an explanation in terms of algebraic geometry, an approach also taken up by Knörrer [1985].

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