Abstract. A random matrix theory has found an extraordinary number of applications in mathematics, statistics, physics or finance. One of particular problems is to study a behaviour of (largest) eigenvalues of a covariance matrix \( XX' \) associated with a data matrix \( X \). The data matrix is defined by \( p \) rows and \( n \) columns, where \( p \) is possibly much larger than the number \( n \). Intuitively, \( p \) can represent the number of different stocks and then \( n \) describes their prices at distinct time points. The objective of this research is to observe the possible limiting distributions for (largest) eigenvalues of large covariance matrices when independence and normality assumptions are dropped. In this project, we will use existing theoretical results for independent data as a benchmark for our extensive Monte Carlo experiments. We simulate a large number of time series and compute empirical distribution of eigenvalues of the associated sample covariance matrix.

2.1 WHAT IS KNOWN?
Assume that each row consists of independent and identically distributed random variables. Furthermore, assume that rows are independent of each other. Then the eigenvalues follow Marchenko-Pastur Law:

\[
L(dx) = \frac{1}{2\pi x} \sqrt{(x_+ - x)(x - x_-)}
\]

where

\[
x_- < x < x_+\]

and

\[
x_\pm = (1 + \sqrt{y})^2, \quad y = \lim_{n} \frac{\rho}{n}.
\]

2.2 Illustration of Marchenko-Pastur Law
We illustrate the Marchenko-Pastur Law for i.i.d. data as a benchmark for comparison with time series models.
- Graph 1: \( n=5000, p=100 \)
- Graph 2: \( n=5000, p=500 \)

2.1 AR(1) model
AutoRegressive model of order 1 is defined as a solution to a recurrence equation

\[
X_t = \rho X_{t-1} + Z_t
\]

where \( |\rho| < 1 \)
Assume that \( Z_t \) is a sequence of i.i.d random variables with mean zero.

2.2 SV model
Stochastic volatility (SV) model is defined as

\[
X_t = \sigma(Y_t)Z_t
\]

where \( \sigma \) is a positive function, \( Y_t \) is stationary sequence, such that \( Y_t \) and \( Z_t \) are independent for given \( i \).

CONCLUSIONS:
- When \( X_t \) are i.i.d. standard normal, then the empirical distribution of the eigenvalues follows Marchenko-Pastur Law. This is best illustrated for \( n = 5000, p = 500 \); see Graph 2.
- Now, consider \( X_t \) to be AR(1), in the same case of \( n = 5000, p = 500 \). When \( \rho = 0.5 \), the histogram has a similar shape as in the i.i.d. case, but with a bigger range of eigenvalues; for \( \rho = 0.8 \), it has a different shape as compared to the i.i.d. case, and also has a bigger range of eigenvalues. This empirical observations suggests that the Marchenko-Pastur Law may not be valid in time series case.
- At last, consider \( X_t \) to be SV, in the same case of \( n = 5000, p = 500 \). When \( \rho = 0.5 \), the histogram has a different shape as compared to the i.i.d. case, and with a bigger range of eigenvalues; the same for \( \rho = 0.8 \). This observation suggests that the Marchenko-Pastur Law may not be valid in stochastic volatility case.