This set of questions covers material from Chapter 1. Multiple choice is the same format as for the midterm.

Q1. A container of 100 light bulbs contains five bad bulbs. We draw 10 bulbs without replacement. Find probability of drawing at least one defective bulb.
   (a) 0.416 (b) 0.584 (c) 0.1 (d) none of the preceding

**Solution to Q1:**

Note: this is not a binomial experiment, just classical probability!!!

Let $X$ - number of defective bulbs. To compute $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$. Now,

$$P(X = 0) = \binom{95}{10} \binom{5}{0} \binom{100}{10} = 0.584.$$ 

Q2. Let $X$ be a discrete random variable with range \{0, 1, 2\} and probability mass function (p.m.f.) given below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The expected value and the variance of $X$ are, respectively:

(a) 0.7, 0.61  (b) 0.7, 1.1  (c) 0.5, 0.61  (d) 0.5, 1.1  (e) none of the preceding.

Q3. A factory employs several thousand workers, of whom 30% are from non-English speaking background. If 15 members of the union executive committee were chosen from the workers at random, evaluate the probability that exactly 3 members of the committee are non-English background people.

(a) 0.17  (b) 0.83  (c) 0.98  (d) 0.51  (e) none of the preceding

**Solution to Q3:**

$X$ - number with non-English background. $X \sim B(15, 0.3)$, to compute $P(X = 3) = 0.17$

Q4. In the situation of Question 3, the probability that majority of the committee has non-English background is

(a) 0.052  (b) 0.83  (c) 0.98  (d) 0.05  (e) none of the preceding

**Solution to Q4:**

$X$ - number with non-English background. $X \sim B(15, 0.3)$, to compute $P(X > 7.5) = P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.95 = 0.05$.

Q5. A player of a video game is confronted with a series of opponents and has an 80% probability of defeating each one. Success with any opponent is independent of previous encounters. The player continues until defeated. The probability that the player plays with at least three opponents, is

(a) 0.8  (b) 0.64  (c) 0.5  (d) 0.36  (e) none of the preceding
Note: You may need to know that for \( q \in (0, 1), \sum_{x=k}^{\infty} q^x = q^k / (1 - q) \).

**Solution to Q5:**

Let \( X \) - number of opponents contested. This is the number of game played, including the last win which is lost. \( X \) is geometric, success = loss - to be consistent with textbook notation, \( p = 0.2 \), i.e. \( P(X = x) = (1 - p)^{x-1}p, \ x = 1, \ldots \). To compute

\[
P(X \geq 3) = \sum_{x=3}^{\infty} (1 - p)^{x-1}p = p \frac{(1 - p)^2}{p} = (1 - p)^2 = 0.64.
\]

**Q6.** In the situation of Question 5, how many opponents is the player expected to play?

(a) 5  (b) 4  (c) 8  (d) 10  (e) none of the preceding

**Solution to Q6:**

Let \( X \) - number of opponents contested. This is the number of game played, including the last win which is lost. \( X \) is geometric, success = loss - to be consistent with textbook notation, \( p = 0.2 \), i.e. \( P(X = x) = (1 - p)^{x-1}p, \ x = 1, \ldots \). To compute

\[
E[X] = \frac{1}{p} = 5
\]

**Q7.** From past experience it is known that 3% of accounts in a large accounting population are in error. The probability that exactly 5 accounts are audited before an account in error is found, is:

(a) 0.242  (b) 0.011  (c) 0.030  (d) 0.026  (e) none of the preceding

**Solution to Q7:**

\( X \sim \text{Geometric.} \)

\[
P(X = 5) = P(1\text{st 4 are correctly stated })P(5\text{th in error}) = 0.97^4(0.03) \approx 0.026
\]

**Q8.** A receptionist receives on average 2 phone calls per minute. Assume that the number of calls can be modeled using a Poisson random variable. What is the probability that there is no calls within 3 minutes?

(a) \( e^{-2} \)  (b) \( e^{-1/2} \)  (c) \( e^{-6} \)  (d) \( e^{-1} \)  (e) none of the preceding

**Solution to Q8:**

We have Poisson random variable \( X \) with \( \lambda = 6/(3 \text{ minutes}) \). To compute: \( P(X = 0) = \lambda^0 \exp(-\lambda) = \exp(-6) \).

**Q9.** Consider a random variable \( X \) with the following p.d.f:

\[
f(x) = \begin{cases} 
0 & \text{if } x \leq -1 \\
\frac{3}{4} (1 - x^2) & \text{if } -1 < x < 1 \\
0 & \text{if } x \geq 1
\end{cases}
\]

Calculate the expected value and the standard deviation of \( X \).

(a) 0 and 3  (b) 0 and .4742  (c) 1 and .2
Q10. Random variable $X$ has a cumulative distribution function given by

$$F(x) = \begin{cases} 
0, & x \leq 0 \\
x/2, & x \in (0, 2) \\
1, & x \geq 2
\end{cases}$$

Its mean value is given by

(a) 1 (b) 2 (c) 0 (d) 0.5 (e) none of above

**Solution to Q10:**

The density is given by $f(x) = \frac{1}{2}$ for $x \in (0, 2)$ and 0 otherwise. Therefore, the mean is $\int_0^2 \frac{1}{2} x dx = 1.$

Q11. Assume that $X$ has density function given by $f(x) = \frac{3}{2} x^2$, $x \in (-1, 1)$. Find $P(X^2 \leq 0.25)$.

(a) 0.250 (b) 0.125 (c) 0.500 (d) 0.061 (e) none of above

**Solution to Q11:**

The distribution function is

$$F(x) = \begin{cases} 
0, & x \leq -1 \\
x^3/2, & x \in (0, 2) \\
1, & x \geq 1
\end{cases}$$

Thus

$$P(X^2 \leq 0.25) = P(-0.5 \leq X \leq 0.5) = F(0.5) - F(-0.5)$$

$$= \frac{0.5^3}{2} - \frac{(-0.5)^3}{2} = 0.125.$$

Q12. In the inspection of tin plate produced by a continuous electrolytic process, 0.2 imperfections are spotted per minute, on average. Find the probability of spotting at least two imperfections in 5 minutes. Assume that we can model the occurrences of imperfections as a Poisson process.

(a) .736 (b) .264 (c) .632 (d) .368 (e) none of the preceding

**Solution to Q12:**

Rate per 5 minutes: $\lambda = 1$. If $X \sim \text{Poisson}, \lambda$, then to compute $P(X \geq 2) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1))$:

$$P(X = 0) + P(X = 1) = \exp(-\lambda) + \exp(-\lambda)\lambda = 0.736.$$

Thus, answer b).

Q13. If $X \sim \mathcal{N}(0, 4)$ the value of $P(|X| \geq 2.2)$ is (use the normal table)

(a) 0.232131 (b) 0.843822 (c) 0.252689 (d) 0.2713 (e) 0.728622

**Solution to Q13:**
\[ P(|X| \geq 2.2) = 1 - P(|X| \leq 2.2) = 1 - P(-2.2 \leq X \leq 2.2) = \\
= 1 - P\left(\frac{-2.2 - 0}{\sqrt{4}} \leq \frac{X - 0}{\sqrt{4}} \leq \frac{2.2 - 0}{\sqrt{4}}\right) = \\
= 1 - (\Phi(1.1) - \Phi(-1.1)) = 0.2713. \]

**Q14.** If \( X \sim \mathcal{N}(10, 1) \), the value of \( k \) such that \( P(X \leq k) = 0.701944 \) is closest to
(a) 0.59 (b) 0.30 (c) 0.53 (d) 10.53 (e) none of the preceding

**Solution to Q14:**

\[ P(X \leq k) = P\left(\frac{X - 10}{1} \leq \frac{k - 10}{1}\right) = P(Z \leq k - 10) = 0.701944 \]

Thus, \( k - 10 = 0.53 \) and \( k = 10.53 \).

**Q15.** The time it takes a supercomputer to perform a task is normally distributed with mean 10 milliseconds and standard deviation 4 milliseconds. What is the probability that it takes more than 18.2 milliseconds to perform the task? (use the normal table)
(a) 0.979818 (b) 0.845632 (c) 0.020182 (d) 0.223578 (e) none of the preceding
Solutions to multiple choice questions:

Q1 → a
Q2 → a
Q3 → a
Q4 → d
Q5 → b
Q6 → a
Q7 → d
Q8 → c
Q9 → b
Q10 → a
Q11 → b
Q12 → b
Q13 → d
Q14 → d
Q15 → c