Chapter 7

Negatively charged exciton on a quantum ring

The last system I am going to consider is the electron-hole system confined in a quantum ring. The results of the treatment of a single electron, an exciton and a negatively charged exciton in the quantum-ring potential and in an external magnetic field can be found in two papers: "Negatively charged exciton on a quantum ring", by Marek Korkusiński, Pawel Hawrylak, and Manfred Bayer, published in Physica Status Solidi (b), vol. 234, page 274 (2002), and "Optical detection of the Aharonov-Bohm effect on a charged particle in a nanoscale quantum ring", by Manfred Bayer, Marek Korkusiński, Pawel Hawrylak, Thomas Gutbrod, M. Michel, and Alfred Forchel, published in Physical Review Letters, vol. 90, page 186801 (2003). Both these publications are integral parts of this thesis and are appended to the presented material.

In Section 2.3 I have described the single-particle energy spectrum of a single electron confined in a quantum ring, and subject to an external magnetic field directed along the axis of rotational symmetry of the system. I have shown that the single-particle ground-state energy exhibits Aharonov-Bohm oscillations as a function of the number of flux quanta threading the ring. In this Chapter I consider the behaviour of an electron-hole pair (a single exciton) and a complex composed of two electrons and one hole (a negatively charged exciton, X^-) confined in such ring. I show that correlations between particles may lead to measurable effects, manifesting themselves in a complete or partial suppression of the Aharonov-Bohm oscillations in the total energy of these systems.

As in Section 2.3, I assume that the ring is infinitesimally thin and narrow, which allows to describe the motion of each particle just by one coordinate (the angle ϕ). The ring is therefore fully characterised by its radius R, and, as I will show, the single-particle energy and the Coulomb interactions scale differently with R: for small rings the single-particle (orbital) energy quantisation dominates, while for large rings the behaviour of the system is strongly affected by interactions. I analyse the properties of the systems in both regimes.

I start the analysis with a single exciton. I write the Schrödinger equation of the system using the centre-of-mass and relative coordinates, which reduces the two-body problem to two single-body problems:

$$\hat{H}_{CM} = -\frac{\hbar^2}{2M_X R^2}\frac{\partial^2}{\partial\phi_{CM}^2}; \tag{7.1}$$

$$\hat{H}_{rel} = \frac{\hbar^2}{2\mu R^2}\left(-i\frac{\partial}{\partial\phi_{rel}} + N_\phi\right)^2 - \frac{e^2}{2R\varepsilon}\frac{1}{\sqrt{d^2+\sin^2(\phi_{rel}/2)}}. \tag{7.2}$$

Here $M_X = m_e + m_h$ is the total mass of the exciton, $\mu = m_e m_h/(m_e + m_h)$ is the reduced mass of the relative particle, and the parameter d is introduced to account for the finite width of the ring and to prevent the Coulomb interactions from diverging. N_ϕ is the number of flux quanta threading the ring; its definition was given in Section 2.3.

As can be seen from Eq. (7.1), the motion of the centre-of-mass particle is not affected by the magnetic field, as is expected, considering the fact that the exciton as a whole is a charge-neutral object. The energy of the centre-of-mass particle does not exhibit the Aharonov-Bohm oscillations, and does not carry any signature of the electron-hole interactions.

On the other hand, the orbital energy operator of the relative particle (first term in Eq. (7.2)) is that of a single charged particle with the reduced mass μ. It conserves the angular momentum k of the relative particle, and via its quadratic dependence on the number of flux quanta N_ϕ it introduces the Aharonov-Bohm oscillations to the energy spectrum of the particle. The relative Hamiltonian (7.2) also contains the Coulomb term, introducing a residual attractive potential of a fixed point charge. This potential mixes the states of the relative particle with different angular momenta, and thus leads to averaging out of the Aharonov-Bohm oscillations in the spectrum. Thus, the behaviour of the particle in the magnetic field depends on the relative strength of these two terms. I study the energy spectrum of the exciton in the regime of both weak and strong interactions using the exact diagonalisation method in the basis of states $\{|k\rangle\}$ of the relative particle, where the angular momentum $k = -10, -9, \ldots, 10$.

For small disks, the single-particle (orbital) energy quantisation dominates and the symmetry breaking introduced by the Coulomb term is weak. The energy of the relative particle, and thus also the total energy of the exciton, exhibits Aharonov-Bohm oscillations. Thus, in this regime, the electron and the hole move almost independently, and the oscillations of each carrier add to produce Aharonov-Bohm oscillations of the total energy of the system.

For large disks, the interactions dominate the energy landscape of the system. The Coulomb term in the relative Hamiltonian strongly couples the configurations with different angular momenta, which causes the oscillations to be averaged out. The total energy of the system changes monotonically with the magnetic field (exhibits the diamagnetic shift). In this regime the electron and the hole are strongly correlated and move as one, charge-neutral object.

In analysing the negatively charged exciton X^-, composed of two electrons and one hole moving on the ring, I use a similar formalism of centre-of-mass and relative coordinates. In these coordinates the Hamiltonian of the system separates into the centre-of-mass Hamiltonian, composed only of the orbital energy operator, and the relative Hamiltonian, describing the motion of two interacting relative particles:

$$\hat{H}_{CM} = \frac{\hbar^2}{2MR^2}\left(-i\frac{\partial}{\partial\phi_{CM}} + N_\phi\right)^2, \tag{7.3}$$

$$\begin{aligned}\hat{H}_{rel} &= \sum_{i=1}^{2}\frac{\hbar^2}{2\mu R^2}\left(-i\frac{\partial}{\partial\phi_i} + N_\phi\right)^2 - \frac{e^2}{2R\varepsilon}\frac{1}{\sqrt{d^2+\sin^2(\phi_i/2)}} \\ &+ \frac{e^2}{2R\varepsilon}\frac{1}{\sqrt{d^2+\sin^2((\phi_1-\phi_2)/2)}} + \frac{\hbar^2}{m_h R^2}\left(-i\frac{\partial}{\partial\phi_1}\right)\left(-i\frac{\partial}{\partial\phi_2}\right) \\ &- \frac{2\sigma}{1+2\sigma}\frac{\hbar^2}{2\mu R^2}N_\phi^2,\end{aligned} \tag{7.4}$$

where $\sigma = m_e/m_h$, and $M = 2m_e + m_h$ is the mass of the entire X^- complex. I solve for the centre-of-mass motion analytically, and use an exact diagonalisation approach to analyse the motion of relative particles.

Since now the entire complex carries an effective negative electron charge, the centre-of-mass particle is charged as well. That is why the centre-of-mass Hamiltonian contains terms dependent upon the magnetic field, and the single-particle (orbital) energy of the centre-of-mass particle exhibits Aharonov-Bohm oscillations. However, the amplitude of these oscillations is small compared to that of a single electron because of the large mass M of the centre-of-mass particle.

As for the two relative particles, they are light and charged, and they move in the additional residual attractive potential of a fixed point charge, similar to the one obtained in the case of the exciton. The particles interact via Coulomb potential and a pairwise momentum interaction, appearing as a result of the transformation of the system's kinetic energy operator into relative coordinates. The last term in the relative Hamiltonian is proportional to N_ϕ^2 and contributes to the diamagnetic shift.

The single-particle orbital energy operators for the relative particles introduce the Aharonov-Bohm oscillations. Both pairwise interaction terms conserve the total angular momentum of the pair, and therefore do not lead to the suppression of these oscillations. However, the residual single-body attractive potential mixes the states of the pair with different angular momenta, and can lead to the suppression of the oscillations, similarly as it did in the case of the single exciton. Quantitative calculations for the pair of relative particles are carried out using the configuration-interaction method, in the basis of configurations $|k_1 k_2\rangle$ created as Slater determinants of single-particle orbitals with definite angular momentum. To optimise the basis, I rotate the set constructed with configurations to the set consisting of eigenvectors of the total spin operator. Since I deal with two particles only, this rotation allows to separate the spin singlet and spin triplet states, and to perform the exact diagonalisation in each subspace separately.

Based on this analysis, I find that the total energy of the X^- complex always oscillates in the magnetic field, but, depending on the disk radius, these oscillations can have larger or smaller amplitude. For small disks the symmetry breaking introduced by the residual potential in the relative Hamiltonian is small, the three particles move almost independently, and their total energy oscillates with a large amplitude. On the other hand, for large disks the configurations of relative particles are strongly mixed, and their oscillations average out. The total energy is modulated only weakly by oscillations of the heavy centre-of-mass particles, and exhibits the diamagnetic shift. In this case, the X^- complex is composed of strongly correlated particles and moves in the magnetic field as one object.

Since both the exciton and the X^- are composed of carriers of both types, the electron-hole recombination process can occur resulting in an emission of one photon, which can be further detected. As I discussed in Section 1.4, the energy of this photon is equal to the difference between the initial state (before the recombination) and the final state of the system (after the recombination). In this way, one can directly measure the energy of the exciton, but in the case of the X^- one will detect the energy difference between the total energy of the complex and the energy of the final-state electron. These energies have indeed been measured experimentally by means of photoluminescence spectroscopic techniques. The ring used in the experiment was large enough for the system to be in the strongly interacting regime, where, as I have explained, the excitonic complexes are expected to behave as strongly correlated entities. This behaviour is indeed seen in the experimental data. The photoluminescence line attributed to the single exciton on the ring exhibits only a diamagnetic shift as a function of the magnetic field while the line attributed to the X^- complex is strongly modulated. Since in this regime the oscillations of the total energy of X^- are almost suppressed, the modulation seen in the experiment can only be due to the final state electron. This is confirmed by the characteristic shape of the photoluminescence trace, similar to the inverted sequence of single-electron oscillations. Thus, one directly observes the Aharonov-Bohm oscillations of a *single* electron, which is possible due to the correlated character of the initial-state X^- complex.