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Université d'Ottawa • University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT1300: Mathematical methods I
Fall 2019
Midterm exam #2

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LASTNAME______________________ Firstname:___________________
(in capital letters)
Student number:___________________

Instructions:

- The duration of this exam is 80 minutes.
- **NO CALCULATORS. NO BOOKS. NO NOTES.**
- This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 40 points.
- For the **multiple choice** questions: write your answer (letter 'A' to 'F') in the table below.
- For the **long answer questions**: write clearly the solution in the space following the problem. You may use the back of any other page if necessary, but you have to clearly indicate the page number where your solution continues.
- Don't detach this examem.
- **NB**
  Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in your obtaining a 0 (zero) for the exam.
  By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature:__________________

Answers:

	1	2	3	4	5	6	7	Total
Problem	multiple choice problems (your answer: a letter A-F)				long solution problems (space for the marker)			
Votre résultat	F	D	A	C				

Multiple Choice Section Questions (1-5)

Question 1 *Suppose* $f'(x) = 1 + \frac{4}{x^2} + \frac{1}{\sqrt{x}}$ *and that* $f(1) = 2$. *Find* $f(4)$.

A) -1 **B)** 1 **C)** $\frac{2}{11}$ **D)** 8 **E)** $\frac{1}{8}$ $\boxed{\textbf{F)}\ 10}$

$$\rightarrow f(x) = \int f'(x)\,dx = \int \left(1 + \frac{4}{x^2} + \frac{1}{\sqrt{x}}\right) dx$$

$$= \int \left(1 + 4x^{-2} + x^{-1/2}\right) dx = x + 4\,\frac{x^{-1}}{-1} + \frac{x^{\frac{1}{2}}}{1/2} + c$$

$$= x - 4x^{-1} + 2x^{1/2} + c$$

$$\rightarrow f(1) = 2: \quad 1 - 4 + 2 + c = 2, \qquad c = 3$$

$$\rightarrow f(4) = 4 - 4 \cdot 4^{-1} + 2 \cdot 4^{1/2} + 3$$

$$= 4 - 1 + 4 + 3 = 10$$

Question 2 *The change rate of the profit function of a company is given in the figure for a period of five years. What is the accumulated (total) profit during the same period?*

A) 200

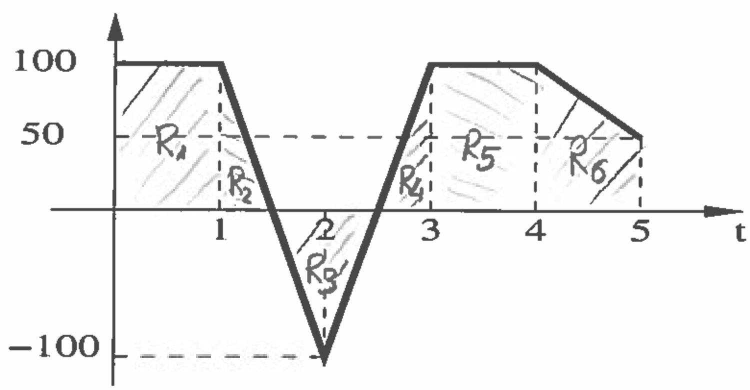
B) 225

C) 250

$\boxed{\textbf{D)}\ 275}$

E) 300

F) 325



$$\rightarrow A = \int_0^5 r(t)\,dt$$

$$= A(R_1) + \left(\cancel{A(R_2)} - \cancel{A(R_3)} + \cancel{A(R_4)}\right) + A(R_5) + A(R_6)$$

$$= 1 \cdot 100 + \left(\qquad 0 \qquad \right) + 1 \cdot 100 + 1 \cdot 50 + \tfrac{1}{2} \cdot 1 \cdot 50$$

$$= 275$$

Question 3 *Suppose that the demand function for a product is given by $p = 14 - 3\sqrt{x}$. What is the elasticity of demand when $x = 9$? Is demand elastic or inelastic?*

$\boxed{\text{A)} \begin{cases} \eta = -\frac{10}{9} \\ \text{elastic} \end{cases}}$ B) $\begin{cases} \eta = \frac{9}{10} \\ \text{inelastic} \end{cases}$ C) $\begin{cases} \eta = -\frac{2}{9} \\ \text{inelastic} \end{cases}$ D) $\begin{cases} \eta = -\frac{10}{30} \\ \text{elastic} \end{cases}$ E) $\begin{cases} \eta = -\frac{9}{5} \\ \text{elastic} \end{cases}$ F) $\begin{cases} \eta = -\frac{7}{10} \\ \text{inelastic} \end{cases}$

$\rightarrow \eta = \dfrac{p(x)}{x \cdot p'(x)} = \dfrac{14 - 3\sqrt{x}}{x \cdot \left(-3 \frac{1}{2\sqrt{x}}\right)}$

$\rightarrow \eta(9) = \dfrac{14 - 3 \cdot 3}{9 \cdot \left(-3 \cdot \frac{1}{2 \cdot 3}\right)} = \dfrac{5}{-9 \cdot \frac{1}{2}} = -\dfrac{10}{9}$

$\rightarrow |\eta(9)| = \frac{10}{9} > 0$, elastic

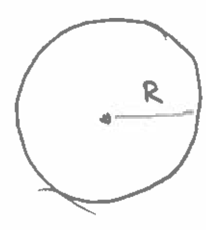
Question 4 *A population of bacteria ocuppies over time a circular region. Its ~~area~~ radius grows at a rate of 3 $\text{km}^{\cancel{2}}$/year. Find the change rate of its area when the radius of the region is 10 km.*

A) 10π B) 30π $\boxed{\text{C) } 60\pi}$ D) 80π E) 100π F) 120π

$\rightarrow A = \pi R^2$ radius

area

$A = A(t)$, $R = R(t)$.



$\rightarrow A' = 2\pi R \cdot R'$

$\rightarrow R' = 3$, $R = 10$

$\rightarrow A = 2 \cdot \pi \cdot 3 \cdot 10 = 60\pi$

Long Answer Questions (6-8)

Question 5 (6 points) *Evaluate*[1] *the following integral:*

$$\int_0^1 \frac{x}{\sqrt{4-x^2}}dx = \int_{x=0}^{x=1} \frac{x}{\sqrt{t}} dx$$

$$t = 4 - x^2$$

$$dt = -2x\,dx, \quad x\,dx = -\tfrac{1}{2}dt$$

$$t(0) = 4, \quad t(1) = 3$$

$$= \int_{x=0}^{x=1} \frac{-\frac{1}{2}dt}{\sqrt{t}}$$

$$= \int_4^3 -\frac{1}{2}\, t^{-1/2}\, dt$$

$$= -\frac{1}{\cancel{2}} \cdot \left[\frac{t^{1/2}}{\cancel{1/2}}\right]_4^3$$

$$= -\left(\sqrt{3} - \sqrt{4}\right)$$

$$= 2 - \sqrt{3}.$$

[1]Show all the details of your work

Question 6 (8 points) *Suppose a function $y = f(x)$ is defined implicitly by the equation*

$$x^2y^2 - 5x^2 - \frac{1}{2}y^2 + 3 = 0$$

near appoint $(-1, 2)$. Find[2] *the derivative $f'(-1)$ and the equation of the tangent line to the graph of f at the point $(-1, 2)$.*

$\rightarrow$ $(x^2y^2 - 5x^2 - \frac{1}{2}y^2 + 3)' = 0$

$2x \cdot y^2 + 2x^2 y y' - 10x - yy' = 0$

$\rightarrow$ $x = -1,\ y = 2$:

$-2 \cdot 2^2 + 2 \cdot 2 \cdot y' + 10 - 2 \cdot y' = 0$

$-8 + 4y' + 10 - 2y' = 0$

$2y' + 2 = 0$

$y' = -1$

$\rightarrow$ tang line eq: at $(x_0, y_0) = (-1, 2)$:

$y = f'(x_0)(x - x_0) + f(x_0)$

$y = -1 \cdot (x - (-1)) + 2$

$y = -x + 1.$

[2]Show all the details of your work

Question 7 (6 points) *Evaluate*[3] *the following integral:*

$$\int xe^{-2x}dx = \int f(x)\, g'(x)\, dx$$

$$= f(x)\cdot g(x) - \int f'(x) g(x) dx$$

$$= -\frac{1}{2} x e^{-2x} - \int 1\cdot\left(-\frac{1}{2} e^{-2x}\right) dx$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2}\int e^{-2x} dx$$

$$= -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C.$$

$f(x) = x$, $\quad f'(x) = 1$

$g'(x) = e^{-2x}$, $\quad g(x) = \int g'(x)\, dx$

$$= \int e^{-2x} dx$$

$$= -\frac{1}{2} e^{-2x}$$

[3] Show all the details of your work