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Department of Mathematics and Statistics   MAT1300B
University of Ottawa   Fall 2018
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+ Sol (a)

LAST name_____   First name:_____
Student number:_____

Instructions:

- The duration of this exam is 80 minutes.
- The use of books, notes or calculators is not allowed.
- For the **multiple choice** questions: write your answer (letter 'A' to 'E') in the table below.
- For the **long answer questions**: write clearly the solution in the space following the problem. You may use the back of any other page if necessary, but you have to clearly indicate the page number when doing so.
- Don't detach this examem.
- **NB**
  Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in your obtaining a 0 (zero) for the exam.
  By signing below, you acknowledge that you have ensured that you are complying with the above statement.

  Signature:_____

Answers:

	1	2	3	4	5	6	7	Total
Problem	multiple choice questions (write a letter A-E)				long solution questions (don't write anything here)			
Your result	B	C	E	A				

Multiple Choice Questions (1-4)

Question 1 *The demand function of a product is given by* $p(x) = 300 - 0.2x$, *where* x *is the quantity and* $p(x)$ *is the price. Compute the elasticity* η *at* $x = 200$ *and find which of the following statements is correct?*

A) $\eta = 26/4$, *inelastic*   **B)** $\eta = 26/4$, *elastic,*   **C)** $\eta = 13/3$, *elastic,*

D) $\eta = 3/13$, *inelastic,*   **E)** $\eta = 260$, *elastic*

$$\eta = \left| \frac{p(x)}{x \cdot p'(x)} \right| = \left| \frac{300 - 0.2x}{x \cdot (-0.2)} \right|$$

$$\eta(200) = \left| \frac{300 - 0.2 \cdot 200}{200 \cdot (-0.2)} \right| = \left| \frac{300 - 40}{-40} \right| = \frac{260}{40} = \frac{26}{4}$$
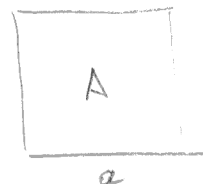
$$= \frac{13}{2}$$

Question 2 *A city grows in the form of a square. It is observed that its side grows at a rate of* 0.5 *km/year. Find the rate change of its area when the side has length* 10 *km.*

A) 0.5   **B)** 5   **C)** 10   **D)** 50   **E)** 100

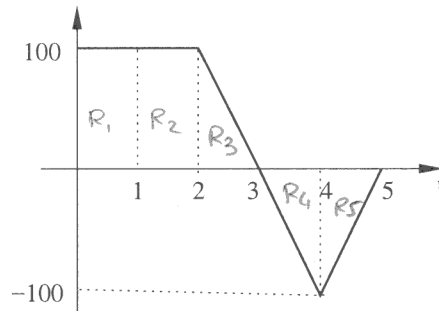$A = a^2$

$A' = 2a \cdot a'$

$A = 2 \cdot 10 \cdot 0.5 = 10$

$A = a^2$

2

Question 3 *The rate change of the balance in a bank account is given in the figure for a period of five years. Find how much money is accommulated in the account during the same period?*

A) 25

B) 50

C) 75

D) 100

E) 150

$$A = \int_0^5 f(t)dt = A(R_1) + A(R_2) + A(R_3) + A(R_4) + A(R_5)$$

$$= 100 + 100 + 50 - 50 - 50$$

$$= 150$$

Question 4 *The function $y = y(x)$ is defined implicitly by the equation*

$$x^2 - 3xy + 7y = 5.$$

Find the derivative $y'(2)$ knowing that $y(2) = 1$.

A) −1   B) 0   C) 1   D) 2   E) 3

$x = 2, \quad y = 1 \quad : \quad 2^2 - 3 \cdot 2 \cdot 1 + 7 \cdot 1 = 4 - 6 + 7 = 5 \quad \checkmark$

$2x - 3y - 3 \cdot x y' + 7y' = 0$

$2 \cdot 2 - 3 - 3 \cdot 2 \cdot y' + 7y' = 0$

$1 + y' = 0$

$y' = -1$

3

Long Answer Questions (5-7)

Question 5 (13 points) *Compute the following integral.*

$$\int \frac{x}{2-x}dx =$$

Solution

$$\int \frac{x}{2-x} dx =$$

$$\begin{cases} t = 2-x, \quad x = 2-t \\ dt = -dx \\ dx = -dt \end{cases}$$

$$= \int \frac{2-t}{t}(-dt)$$

$$= \int \left(-\frac{2}{t} + 1\right) dt$$

$$= -2\ln|t| + t + C$$

$$= (2-x) - 2\ln|2-x| + C$$

Question 6 (15 points) *Let $f(x) = \sqrt{x}$.*
i) Let $x_0 = \frac{9}{4}$ and find $f(x_0)$.
ii) Find the equation of the tangent line equation to the graph of f at the point $(x_0, f(x_0))$.
iii) Using the tangent line approximation at $(x_0, f(x_0))$ find an approximation of $\sqrt{2}$.

Solution

i) $f\left(\frac{9}{4}\right) = \sqrt{\frac{9}{4}} = \frac{3}{2}$

ii) Eq. of tang. line at $(x_0, f(x_0))$ is:

$$y = f'(x_0)(x - x_0) + f(x_0);$$

Here: $\quad x_0 = \frac{9}{4}, \quad f(x_0) = \frac{3}{2},$

$$f'(x_0) = \frac{1}{2\sqrt{x_0}} = \frac{1}{2\sqrt{\frac{3}{4}}} = \frac{1}{2 \cdot \frac{3}{2}} = \frac{1}{3}, \qquad \text{because } f'(x) = \frac{1}{2\sqrt{x}}.$$

Hence:

$$y = \frac{1}{3}\left(x - \frac{9}{4}\right) + \frac{3}{2}$$

$$= \frac{1}{3}x - \frac{1}{3} \cdot \frac{9}{4} + \frac{3}{2} = \frac{1}{3}x - \frac{3}{4} + \frac{3}{2}$$

$$y = \frac{1}{3}x + \frac{3}{4}$$

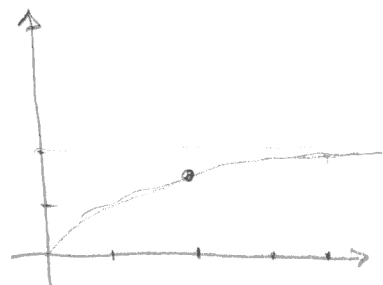iii) We approximate $\sqrt{2}$ by the tangent line equation at $\left(\frac{9}{4}, \frac{3}{2}\right)$ because $\frac{9}{4} = 2 + \frac{1}{4}$, so close to 2; then.

$$\sqrt{2} \approx \frac{1}{3} \cdot 2 + \frac{3}{4} \qquad \left[\text{this is tang. line eq for } x = 2\right]$$

$$= \frac{2}{3} + \frac{3}{4} = \frac{8 + 9}{12}$$

$$= \frac{17}{12}$$

$$= 1.4166\ldots$$

5

Question 7 (12 points) *The demand function of a product is $p(x) = -3x + 7$, and its supply function is $s(x) = 2x + 2$.*
i) Find the equilibrium demand and equilibrium price of this product.
ii) Find the conusmer and producer surpluses.

Solution

i) Equilibrim point:

$$p(x) = s(x)$$

$$-3x + 7 = 2x + 2$$

$$5x = 5, \qquad x_0 = 1, \qquad p_0 = p(x_0) = -3 \cdot 1 + 7 = 4$$

$x_0 = 1, \quad p_0 = 4.$

ii) $C.S. = \int_0^1 (-3x + 7 - 4) \, dx$

$$= \left[-3 \cdot \frac{1}{2} x^2 + 3x \right]_0^1$$

$$= -\frac{3}{2} + 3$$

$$= \frac{3}{2},$$

$P.S. = \int_0^1 (4 - 2x - 2) \, dx$

$$= \left[2x - x^2 \right]_0^1$$

$$= 2 - 1$$

$$= 1.$$