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Mathematical methods I

MAT1300B
Fall 2017
Midterm exam #1

LAST name________________________ First name:____________________
Student number:____________________

Instructions:

- The duration of this exam is 80 minutes.
- The use of books, notes or calculators is not allowed.
- For the **multiple choice** questions: write your answer (letter 'A' to 'E') in the table below.
- For the **long answer questions**: write clearly the solution in the space following the problem. You may use the back of any other page if necessary, but you have to clearly indicate the page number when doing so.
- Don't detach this examem.
- **NB**
  Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in your obtaining a 0 (zero) for the exam.
  By signing below, you acknowledge that you have ensured that you are complying with the above statement.

  Signature:________________

Answers:

	1	2	3	4	5	6	7	Total
Problem	multiple choice (write a letter A-E)				long solution (don't write anything here)			
Votre résultat	B	E	B	D.				

Multiple Choice Section Questions (1-4)

Question 1 *For what value of k is the following function continuous for all x?*

$$f(x) = \begin{cases} k - x & \text{if } x < 1, \\ \dfrac{3}{k + x^2} & \text{if } x \geq 1. \end{cases}$$

A) 8 **B)** $-2, 2,$ **C)** $-3, 3,$ **D)** $2, 3,$ **E)** 0

$\to \quad k-1 = \dfrac{3}{k+1}, \quad (k-1)(k+1) = 3$

$k^2 - 1 = 3$

$k^2 = 4, \quad k = \pm 2$

Question 2 *Find the domain of definition of $f(x) = \sqrt{\frac{x}{1-x}}$.*

A) $(-\infty, 0] \cup (1, \infty)$ **B)** $(0, 1)$ **C)** $[0, 1]$ **D)** $(0, 1]$ **E)** $[0, 1)$

$\to \quad \dfrac{x}{1-x} \geq 0, \quad x \neq 1.$

x		0		1	
x	$-$	0	$+$		$+$
$1-x$	$+$		$+$	0	$-$
$f(x)$	$-$	0	$+$	‖	$-$

$\to \quad [0, 1).$

Question 3 *Solve the following equation for x.*

$$3^{x-1} = 2^{2-x}.$$

A) $\ln(9)$ B) $\boxed{\frac{2\ln(2)+\ln(3)}{\ln(2)+\ln(3)}}$ C) $\frac{2\ln(3)}{\ln(2)+\ln(3)}$ D) $\frac{\ln(9)}{2\ln(3)}$ E) $\frac{3\ln(3)}{\ln(9)-\ln(2)}$

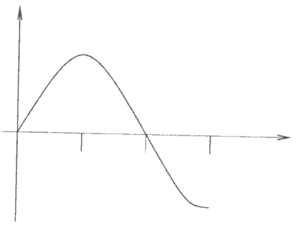
$\rightarrow \quad \ln(3^{x-1}) = \ln(2^{2-x})$

$(x-1)\ln 3 = (2-x)\ln 2$

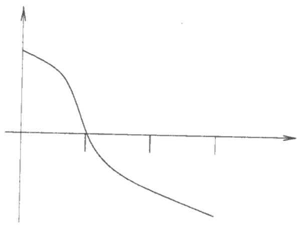
$x\cdot(\ln 3 + \ln 2) = 2\ln 2 + \ln 3$

$$x = \frac{2\ln 2 + \ln 3}{\ln 2 + \ln 3} = \frac{\ln(2^2\cdot 3)}{\ln(2\cdot 3)} = \frac{\ln(12)}{\ln 6}.$$

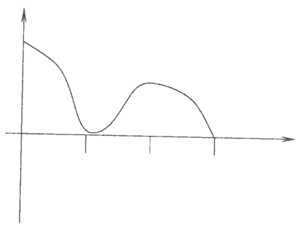
Question 4 *The picture f) below is the graph of a function f. Find which of the pictures A)-E) represents the graph of f'.*



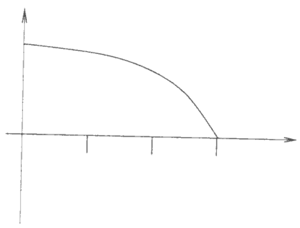
f)



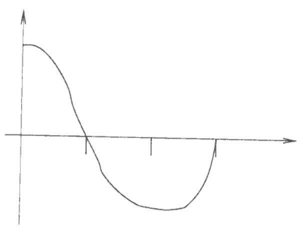
A)



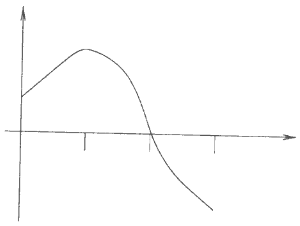
B)



C)



D)



E)

Long Answer Section Questions (5-7)

Question 5 (14 points) *Let $f(x) = \sqrt{2-x}$. Find $f'(0)$ by using only the definition of the derivative as a limit.*

Sol

3 $$f'(0) = \lim_{h \to 0} \frac{1}{h}\left(f(0+h) - f(0)\right)$$

3 $$= \lim_{h \to 0} \frac{1}{h}\left(\sqrt{2-h} - \sqrt{2}\right)$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{\left(\sqrt{2-h} - \sqrt{2}\right)\left(\sqrt{2-h} + \sqrt{2}\right)}{\sqrt{2-h} + \sqrt{2}}$$

4 $$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{\left(\sqrt{2-h}\right)^2 - \left(\sqrt{2}\right)^2}{\sqrt{2-h} + \sqrt{2}}$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}} \cdot \frac{2 - \cancel{h} - 2}{\sqrt{2-h} + \sqrt{2}}$$

4 $$= -\lim_{h \to 0} \frac{1}{\sqrt{2-h} + \sqrt{2}}$$

$$= -\frac{1}{2\sqrt{2}}.$$

Question 6 (12 points) *The cost function of a product is given by* $C(x) = x^4 + \frac{4}{x}$.
i) What is the marginal cost at $x = 1$*?*
ii) Find the equation of the tangent line to the graph of function C *at the point* $(1, 5)$.

Sol

4 $\rightarrow \quad c'(x) = 4x^3 + 4(x^{-1})'$

$= 4x^3 - 4 \cdot x^{-2}$

2. $c'(1) = 4 - 4 = 0.$

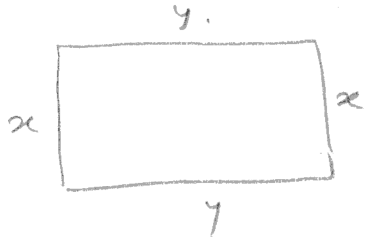
6 $\rightarrow \quad y = c'(1)(x-1) + 5$

$y = 0 \cdot (x-1) + 5$

$y = 5.$

Question 7 (14 points) *The manager of a garden store wants to build a 2400 m^2 rectangular enclosure in order to display some equipment. Three sides of the enclosure will be built of wood fencing, at a cost of 10\$/meter, while the other side will be build of concrete blocks, at a cost of 20\$/meter. Find the dimensions of least cost of a such enclosure.*

Sol



2 → $A = x \cdot y = 2400$

2 → $C = 10(x + x + y) + 20y$

$C = 20x + 30y$

→ From $x \cdot y = 2400$, $y = \dfrac{2400}{x}$, so

3 $C = C(x) = 20x + \dfrac{30 \cdot 2400}{x}$

To find x (and y), $x > 0$, s.th C is minimal.

→ $C'(x) = 20 - \dfrac{30 \cdot 2400}{x^2} = 20\,\dfrac{x^2 - 30 \cdot 120}{x^2}$

3 $C'(x) = 0 \iff x^2 - 30 \cdot 120 = 0$

$x^2 = 3 \cdot 3 \cdot 400$, $x = 3 \cdot 20 = 60$, $\cancel{x = -60}$

2 →

x	0		60		$+\infty$
$C'(x)$		$-$	0	$+$	
$C(x)$		$\searrow$		$\nearrow$	

loc. mini

or $C''(x) = + \dfrac{30 \cdot 2400 \cdot 2}{x^3} > 0$

so loc. mini

1 → as the $x = 60$ is the only C.P. of C in $(0, +\infty)$, $C(60)$ is global minimum of C.

1 → $x = 60$, $y = \dfrac{2400}{60} = 40$ minimize the cost.