University of Ottawa
Department of Mathematics and Statistics

MAT 1341C: Introduction to Linear Algebra
Instructor: Erhard Neher

Test 3 (March 31, 2012)

Family name (CAPITALS)	
First name (CAPITALS)	
Signature	
Student number	

The last three digits of my student number are

third last digit $\alpha =$
second last digit $\beta =$
last digit $\gamma =$

Please read these instructions carefully:

- The table below is for the TA. Do not write in it.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1–2	3	4	5	6	Total
Score						
Max. score	13	6	6	6	(2 bonus)	31

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 3 – 6 you may use the back of pages if necessary, but be sure to indicate to the marker that you have done so. For these questions you must supply all details of your work.
- No part marks will be given for questions 1 and 2. They questions do not require a justification.
- Question 6 is a bonus proof question. You can get 2 extra points.
- No books or notes are allowed. **Calculators, cell phones or any electronic devices are not permitted.**

Good luck! Bonne chance!

(1) (6 pts) Are the following statements true or false? Answer with T for "true" and F for "false".

(a) One calls vectors $X_1, X_2, \ldots, X_k$ linearly independent if the following condition holds: Whenever all scalars $s_1, \ldots, s_k$ are zero then $s_1 X_1 + s_2 X_2 + \cdots s_k X_k = 0$.

My answer:__________________

(b) Any linearly independent subset of $\mathbb{R}^n$ is orthogonal.

My answer:__________________

(c) Every set of 3 vectors in $\mathbb{R}^n$ is a spanning set of a three-dimensional subspace of $\mathbb{R}^n$.

My answer:__________________

(d) If A is an $n \times n$ matrix whose columns span $\mathbb{R}^n$, then A^T is invertible.

My answer:__________________

(e) Any 3 vectors in $\mathbb{R}^2$ will be linearly dependent.

My answer:__________________

(f) If X_1, X_2 are linearly independent and Y is not a linear combination of X_1, X_2, then $\{X_1, X_2, Y\}$ can be linearly dependent.

My answer:__________________

(2) (7 pts) (a) (1 pt) 1 Assume that A is a 4×5 matrix for which the associated homogeneous linear system $AX = 0$ has 3 basic solutions. Then the dimension of the row space of A equals ... (complete the sentence).

My answer:_________________

(b) (2 pts) Let $U \subset \mathbb{R}^6$ be a 4-dimensional space. Then ... (give the dimensions)

$\dim U^{\perp} =$ ______________ and $\dim U^{\perp\perp} =$ ______________

(c) (1 pt) 1 Suppose X_1, X_2, X_3 is a basis of a subspace U in $\mathbb{R}^5$. Then the orthogonal basis F_1, F_2, F_3 of U obtained from X_1, X_2, X_3 by the Gram-Schmidt algorithm is ... (give the formula)

(d) (3 pts) Let α, β, γ be the third last, second last and last digit of your student number. Which one(s) of the following subsets of $\mathbb{R}^3$ is (are) a basis of $\mathbb{R}^3$?

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix} \right\}, \quad B_2 = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \beta \\ 1 \end{bmatrix} \right\}, \quad B_3 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ \gamma \end{bmatrix} \right\}$$

No justification required.

My answer:_________________

(3) (6 pts) Let $U = \operatorname{Span}\{X_1, X_2, X_3, X_4\}$ where the vectors $X_1, \dots, X_4 \in \mathbb{R}^4$ are

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \quad X_4 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 2 \end{bmatrix}.$$

Find a basis of U and $\dim U$.

(4) (6 pts) Let $X_1 = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$ and $X_2 = \begin{bmatrix}1 & -1 & 0\end{bmatrix}^T$.

(a) (2 pts) Show that $\{X_1, X_2\}$ is an orthogonal set. Is it also orthonormal?

(b) (4 pts) Let $U = \text{Span}\{X_1, X_2\}$. Find the point in U closest to $X = \begin{bmatrix}1 & 2 & \alpha\end{bmatrix}^T$ where α is the **third last** digit of your student number.

(5) (a) (3 pts) Let $\mathbb{F}[0,9]$ be the vector space of functions $f\colon [0,9] \to \mathbb{R}$. In the following definition replace β with the **second last** digit of your student number. Show that

$$U = \{f \in \mathbb{F}[0,9] : f(\beta) = 0\}$$

is a subspace of $\mathbb{F}[0,9]$.

(b) (3 pts) Let s, c, h be the functions in $\mathbb{F}[\mathbb{R}]$ defined by

$$s(x) = \sin^2(x), \quad c(x) = \cos^2(x), \quad h(x) = x.$$

Is $h \in \operatorname{Span}\{s, c\}$?

In both parts you must include all details to get full marks.

(6) (2 bonus points) Let v_1 and v_2 be elements of a vector space V. Show that Span $\{v_1, v_2\}$ is a subspace of V. Recall that Span $\{v_1, v_2\}$ is the set of all linear combinations of v_1, v_2. In other words, Span $\{v_1, v_2\} = \{s_1 v_1 + s_2 v_2 : s_1, s_2 \in \mathbb{R}\}$.