University of Ottawa
Department of Mathematics and Statistics

MAT 1341C: Introduction to Linear Algebra

Instructor: Erhard Neher

Test 2 (March 3, 2012)

FAMILY NAME (CAPITALS)	______________
FIRST NAME (CAPITALS)	______________
Signature	______________
Student number	______________

The last three digits of my student number are

third last digit $\alpha =$
second last digit $\beta =$
last digit $\gamma =$

Please read these instructions carefully:

- The table below is for the TA. Do not write in it.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1–2	3–4	5–6	7	8	9	Total
Score							
Max. score	4	4	4	7	6	2 (bonus)	25

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First name (CAPITALS)	______________________________

Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 7 and 8 you may use the back of pages if necessary, but be sure to indicate to the marker that you have done so. For questions 7 and 8 you must supply all details of your work.
- No part marks will be given for questions 1 – 6. These questions do not require a justification.
- Question 9 is a bonus proof question. You can get 2 extra points.
- No books or notes are allowed. **Calculators, cell phones or any electronic devices are not permitted.**

Good luck! Bonne chance!

(1) Replace β be the **second last** digit of your student number. Find the scalar equation of the line parallel to $\begin{bmatrix}1 & 0 & \beta\end{bmatrix}^T$ and passing through $P(3, -2, 5)$.

My answer:____________________________

(2) Let γ be the **last digit** of your student number, find the equation of the plane passing through the points $P(1, 0, -1)$ and perpendicular to the line $\begin{bmatrix}x & y & z\end{bmatrix}^T = \begin{bmatrix}11 & 3 & -1\end{bmatrix}^T + t\begin{bmatrix}1 & \gamma & -3\end{bmatrix}^T$.

My answer:____________________________

(3) Let α be the **third last** digit of your student number. Given two vectors $\overrightarrow{v} = \begin{bmatrix} 2 & \alpha & 5 \end{bmatrix}^T$ and $\overrightarrow{w} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$, write $\overrightarrow{v} = \overrightarrow{v_1} + \overrightarrow{v_2}$ such that $\overrightarrow{v}_1$ is parallel to $\overrightarrow{w}$ and $\overrightarrow{v_2}$ is perpendicular to $\overrightarrow{w}$.

My answer: $\overrightarrow{v_1} =$__________ **My answer:** $\overrightarrow{v_2} =$__________

(4) Let β the **second last digit** of your student number. If β is odd, do question (a), otherwise do question (b):

(a) Let

$$\vec{u} = \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix}; \quad \vec{v} = \begin{bmatrix} 2\beta \\ 4 \\ -2 \end{bmatrix}; \quad \vec{w} = \begin{bmatrix} -2 \\ 1 \\ -\beta \end{bmatrix}$$

Find the volume of the parallelepiped determined by $\vec{u}$, $\vec{v}$ and $\vec{w}$.

(b) Find the area of the triangle with vertices:

$$A = \begin{bmatrix} -1 \\ \beta \\ 3 \end{bmatrix}; \quad B = \begin{bmatrix} -\beta \\ 1 \\ 3 \end{bmatrix}; \quad C = \begin{bmatrix} -\beta \\ \beta \\ -5 \end{bmatrix}$$

My answer:____________________________

(5) Let γ be the **last digit** of your student number. Write $\frac{\gamma+2i}{2+i}$ in the form $a + bi$ with a, b real numbers.

My answer:____________________________

(6) Let α be the **third last** digit of your student number. Find the complex eigenvalues of the matrix

$$\begin{bmatrix} 1 & 2 \\ -(\alpha+1) & -1 \end{bmatrix}.$$

My answer:____________________________

(7) The characteristic polynomial of the matrix A below is $(x-1)^2(x+2)$.

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$$

(a) (1 pt) Find all eigenvalues of A.

(b) (4 pts) For each eigenvalue find the corresponding eigenspace and basic eigenvectors.

(c) (2 pts) Decide if your matrix A is diagonalizable or not. If yes, give an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$. (Justify your answer!)

(8) (6 pts) If the last digit of your student number is odd, work on A_{odd}. Otherwise, choose A_{even}. Find the inverse of the following matrix.

$$A_{odd} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -2 \\ 0 & -1 & 1 \end{bmatrix}, \quad A_{even} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 2 & -2 \\ -1 & 1 & -1 \end{bmatrix}.$$

Show all details of your work.

(9) (2 bonus points) Let

$$\vec{u} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix};$$

Use the general formula for $\vec{u} \times \vec{v}$ to show that the cross product $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$.