

University of Ottawa
Department of Mathematics and Statistics

MAT 1341C: Introduction to Linear Algebra

Instructor: Erhard Neher

Test 1 (Jan. 28, 2012)

FAMILY NAME (CAPITALS)	_____
FIRST NAME (CAPITALS)	_____
Signature	_____
Student number	_____

Please read these instructions carefully:

- The table below is for the TA. Do not write in it.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1.	2	3	4	5	6	7	Total
Score								
Max. score	2	2	3	2	2	5	8	24

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 6 and 7 you may use the back of pages if necessary, but be sure to indicate to the marker that you have done so.
- No part marks will be given for questions 1 – 5.
- No books or notes are allowed. **Calculators, cell phones or any electronic devices are not permitted.**

Good luck! Bonne chance!

- (1) (2 pts) In the matrix below replace α by the **last digit** of your student number. Find the matrix A satisfying the following equation:

$$\left(A^T + 2 \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & -2 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 3 & \alpha \\ -1 & 0 & 2 \end{bmatrix}$$

My answer: $A =$ _____

(2) (2 pts) Consider the matrices

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 3 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 & 2 & -5 & 0 & 3 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & 1 & 2 & -3 & 5 \\ 0 & 0 & 3 & 0 & -2 \\ 1 & 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 & 2 & -1 & 3 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & -6 \end{bmatrix}.$$

Which one is or which ones are in *reduced* row-echelon form?

My answer: _____

- (3) (3 pts) (a) A linear system with 4 variables and 5 equations always has infinity many solutions. True or False? (No justification required).

My answer:_____

- (b) If the linear system $AX = B$ has no solution, any row-echelon form of the augmented matrix of the system has a row of zeros. True or False? (No justification required).

My answer:_____

- (c) If a linear system has 5 equations and 4 variables, the rank of the augmented matrix is at most equal to ... (no justification required):

My answer:_____

- (4) (2 pts) Let A and B be two 3×4 matrices. Which of the following formulas is correct? The (23)-entry of $A^T B = (c_{ij})$ is
- (A) $c_{23} = a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{32} + a_{24}b_{42}$
 - (B) $c_{23} = a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{32}$
 - (C) $c_{23} = a_{21}b_{31} + a_{22}b_{32} + a_{23}b_{32} + a_{24}b_{34}$
 - (D) $c_{23} = a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{32} + a_{24}b_{34}$
 - (E) $c_{23} = a_{12}b_{13} + a_{22}b_{23} + a_{32}b_{33}$
 - (F) $c_{23} = a_{12}b_{12} + a_{22}b_{23} + a_{32}b_{32} + a_{42}b_{42}$

My answer: _____

- (5) (2 pts) Let A and B be matrices. True or false?
(a) If AB is defined, then also BA is defined.

(b) If A^2 can be formed, then A must be square.

My answer:_____

My answer:_____

(6) (5 pts) Solve the following system and write the solution as linear combination of basic solutions:

$$\begin{array}{ccccrc} x_1 & +2x_2 & -x_3 & +x_4 & = & 0 \\ -x_1 & -x_2 & & +2x_4 & = & 0 \\ x_1 & +x_2 & & -2x_4 & = & 0 \end{array}$$

(7) (8 pts) Consider the system of linear equations

$$\begin{aligned}x + 2y + z &= -1 \\ -x - y + pz &= 4 \\ 3x + 4y - z &= q\end{aligned}$$

where p and q are real numbers.

- (a) (6 pts) determine the values of p and q for which the system has
- (i) a unique solution,
 - (ii) no solution,
 - (iii) infinitely many solutions.
- (b) (2 pts) In case (iii) above give all solutions.