University of Ottawa
Department of Mathematics and Statistics

MAT 1341C: Introduction to Linear Algebra
Instructor: Erhard Neher

Assignment 2 (due March 20, 2012, 13:00)

Family name (CAPITALS)	____________________
First name (CAPITALS)	____________________
Signature	____________________
Student number	____________________

The last two digits of my student number are

second last digit $\beta =$
last digit $\alpha =$

Please read these instructions carefully:

- The table below is for the TA. Do not write in it.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1	2	3	4	5	6 (bonus)	Total
Score							
Max. score	6	3	6	6	9	2 (bonus)	30

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question.

Good luck! Bonne chance !

(1) (6 pts) Let α be the last digit of your student number. Which of the following is (are) subspaces? Support your answer with details.

(A) $U_1 = \left\{ \begin{bmatrix} x & y & z \end{bmatrix}^T : x \neq 0 \right\}$,

(B) $U_2 = \{X \in \mathbb{R}^3 : AX = 3X\}$, where A is a 3×3 matrix.

(C) $U_3 = \{\begin{bmatrix} a & b & c \end{bmatrix}^T : a + 2b + c = 3\alpha\}$

(2) (3 pts) Let β be the **second last** digit of your student number. Determine whether the following set is linearly independent. Support your answer with details.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \beta \\ 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} \right\}$$

(3) (6 pts) Let α the last digit of your student number, and let

$$\vec{u} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 2 \\ \alpha \\ \alpha \end{bmatrix} \quad \vec{z} = \begin{bmatrix} -4\alpha - 1 \\ -\alpha \\ 2\alpha + 1 \end{bmatrix}$$

(a) Is $\vec{w} \in \text{Span}\{\vec{u}, \vec{v}\}$? (b) Is $\vec{z} \in \text{Span}\{\vec{u}, \vec{v}\}$?

(4) (6 pts) Let α the last digit of your student number and let β be the second last digit of your student number. We consider

$$E = \left\{ \begin{bmatrix} \alpha s & \alpha s - \beta t & \alpha s + \beta t & -\beta t \end{bmatrix}^T \in \mathbb{R}^4 : s \text{ and } t \in \mathbb{R} \right\}$$

(a) Show that E is a subspace of $\mathbb{R}^4$.
(b) Find a basis of E.
(c) Calculate the dimension of E.

(5) (9 pts) In the matrix A below replace α by the **last digit** and β by the **second last** digit of your student number.

$$A = \begin{bmatrix} 1 & 1 & 0 & \beta \\ 1 & 1-\alpha & -1 & \beta-1 \\ 2 & 2-2\alpha & -2 & 2\beta-2 \\ 3 & 3+2\alpha & 2 & 3\beta+2 \end{bmatrix}.$$

(a) Find the rank of A.
(b) Find a basis for $row(A)$ and the dimension of $row(A)$.
(c) Find a basis for $col(A)$ and the dimension of $col(A)$.
(d) Find a basis of the space of solutions of the homogeneous linear system $AX = 0$.

(6) (2 bonus points) Use the theorems in sections 4.1 – 4.3 to prove the following: Let $U \subset \mathbb{R}^n$ be a non-zero subspace, spanned by some subset $S \subset U$ and let $L \subset S$ be a linearly independent set. Then there exists a basis B of U satisfying $L \subset B \subset S$.