Page 1

University of Ottawa
Department of Mathematics and Statistics

MAT 1341C: Introduction to Linear Algebra
Instructor: Erhard Neher

Test 1

Family name (CAPITALS)	
First name (CAPITALS)	
Signature	
Student number	

Please read these instructions carefully:

- The table below is for the TA. Do not write in it.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1	2	3	4	5	6	Total
Score							
Max. score	4	4	4	8	8	2 bonus	28

University of Ottawa
Department of Mathematics and Statistics
MAT 1341C: Introduction to Linear Algebra
Test 1

Family name (CAPITALS)	_____
First name (CAPITALS)	_____

Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 4 and 5 you may use the back of pages if necessary, but be sure to indicate to the marker that you have done so.
- No part marks will be given for questions 1 – 3.
- Question 6 is a bonus definition-proof question. You can get 2 extra points.
- No books or notes are allowed. **Calculators, cell phones or any electronic devices are not permitted.**

Good luck! Bonne chance!

(1) (a) (1 pt) A linear system with 4 variables and 5 equations always has infinitely many solutions. True or false? (No justification required)

My answer:_____

(b) (1 pt) If the linear system $AX = B$ has no solution, any row-echelon form of A has a row of zeros. True or false? (No justification required)

My answer:_____

(c) (1 pt) If the linear system has 5 equations and 4 variables, the rank of the augmented matrix is at most (no justification required):

My answer:_____

(d) (1 pt) We consider the homogeneous linear system $AX = 0$. If a row-echelon form of A has a row of zeros, the system has a non-trivial solution. True or false? (No justification required)

My answer:_____

(2) (a) (1 pt) Let α be the **last digit** of your student number. Find the matrix A if

$$\left(2A^T + \begin{bmatrix}\alpha \\ 8\end{bmatrix}\right)^T = 4\begin{bmatrix}-1 & 9\end{bmatrix}$$

My answer:_____

(b) (1 pt) If A is a 3×7 matrix, then A and its transpose A^T have the same main diagonal. True or false?

My answer:_____

(c) (1 pt) Let A be a $m \times n$ matrix and let X be a n-column vector. If AX has a zero entry, then A has a row of zeros. True or false?

My answer:_____

(d) (1 pt) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection in the y-axis. The standard matrix of this linear transformation is:

My answer:_____

(3) (a) (1 pt) Consider the following matrices:

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & -5 & 0 & 3 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & 1 & 2 & -3 & 5 \\ 0 & 0 & 3 & 0 & -2 \\ 1 & 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & -1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & -6 \end{bmatrix}.$$

Which one is or which ones are in reduced row-echelon form?

My answer:_____

(b) (3 pts) Complete the theorem below by stating 3 conditions which are equivalent to, but not the same as the condition in (a) of the theorem.

Theorem. *For a $n \times n$ matrix A the following conditions are equivalent:*

(a) *A is invertible.*

(b)

(c)

(d)

Remark: The theorem stated in class had more equivalent conditions. But you are only asked to list 3 of them.

(4) For the system of linear equations

$$\begin{array}{rcrcrcr} x & + & 2y & + & 4z & = & 2 \\ 2x & + & 7y & + & 8z & = & -2 \\ x & + & ay & + & a^2z & = & a \end{array}$$

(a) (6 pts) determine the values of a for which the system has
   (i) no solution,
   (ii) infinitely many solutions,
   (iii) a unique solution.

(b) (2 pts) In case (ii) above describe give all solutions.

(5) (a) (6 pts) In the matrix below **replace β with the second-last digit of your student number** and find its inverse:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & \beta \\ 2 & 4 & -1 \end{bmatrix}$$

(b) (2 pts) Check your answer by verifying $AA^{-1} = I_3$.

(6) (2 bonus points) (a) Give the definition for a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$.
(b) Let A be an $n \times m$ matrix. Show that the transformation $T : \mathbb{R}^n \to \mathbb{R}^m$, given by $T(X) = AX$ is linear.