# University of Ottawa
# Department of Mathematics and Statistics

## MAT 1341C: Introduction to Linear Algebra
## Instructor: Erhard Neher

### Assignment 3: due Thursday, March 4, 2010, 11:30 in the classroom

(version February 23, 2010)

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|---|---|
| Family name (CAPITALS) | ______________________ |
| First name (CAPITALS) | ______________________ |
| Signature | ______________________ |
| Student number | ______________________ |

**Please read these instructions carefully:**

- The table below is for the TA. Do not write in it.
- The assignment has to be submitted with the two cover pages.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

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| Question | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Score | | | | | | |
| Max. score | 6 | 5 | 6 | 5 | 12 | 34 |

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**Good luck! Bonne chance !**

(1) (6 pts) Use cofactor expansion and/or the properties of determinants to evaluate the following determinants:

$$(i)\ \begin{vmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 3 & -1 & 2 & 3 \\ 3 & 1 & 6 & 1 \end{vmatrix} \quad \text{and} \quad (ii)\ \begin{vmatrix} 0 & b & c & d \\ b & 0 & d & c \\ c & d & 0 & b \\ d & c & b & 0 \end{vmatrix}$$

(2) (5 pts) Find a polynomial $p$ of degree 3 satisfying

$$p(-2) = 49, \quad p(-1) = 10, \quad p(1) = -2, \quad p(2) = -23.$$

(3) (6 pts) Find all eigenvalues of the matrix

$$\begin{bmatrix} 5 & 0 & 1 \\ 2 & 4 & 2 \\ 1 & 0 & 5 \end{bmatrix}$$

For each eigenvalue find a basis of the corresponding eigenspace. Finally, decide if $A$ is diagonalizable or not. If yes, give an invertible matrix $P$ and a diagonal matrix $D$ such that $P^{-1}AP = D$. Justify your answer.

(4) (5 pts) We consider the linear dynamical system $V_k = A^k V_0, k = 1, 2, \ldots$ with

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 9 \end{bmatrix}, \quad V_0 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Find (i) a formula for $A^k$, $k = 1, 2, \ldots$, and (ii) a general expression (= formula) for $V_k$.

(5) (12 pts) Answer the following questions. Do not forget to justify your answer.

(a) If $\det(A) = -1$ and $\det(3A) = -27$, what can be said about the size of the matrix $A$?

(b) Find all values of $x$ such that the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 3 & x \\ 1 & 1 & 9 & x^2 \\ 1 & -1 & 27 & x^3 \end{bmatrix}$$

is invertible.

(c) Every $2 \times 2$ matrix has a real eigenvector. True or false?

(d) If $A$ and $C$ are matrices such that $\det A \neq 0$ and $\det(CA) = 0$, then the system $CX = 0$ has a nontrivial solution.

(e) Let $A$ be an $n \times n$ matrix. If $\det A = 0$, then $A$ has an eigenvector.

(f) Let $A$ be a diagonalizable $n \times n$ matrices. The characteristic polynomial of $A$ can be written as product of $n$ linear factors of the form $x - \lambda_i$ with $\lambda_i \in \mathbb{R}$. True or false?