University of Ottawa
Department of Mathematics and Statistics

MAT 1341C : Introduction to Linear Algebra
Instructor : Erhard Neher

Assignment 2 : due Feb. 2, 2010, 13:00 in the classroom

FAMILY NAME (CAPITALS) ______________________________

FIRST NAME (CAPITALS) ______________________________

Signature ______________________________

Student number ______________________________

Please read these instructions carefully :

- The table below is for the TA. Do not write in it.
- The assignment has to be submitted with the two cover pages.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1	2	3	4	5	6	Total
Score							
Max. score	8	10	8	4	7	6	43

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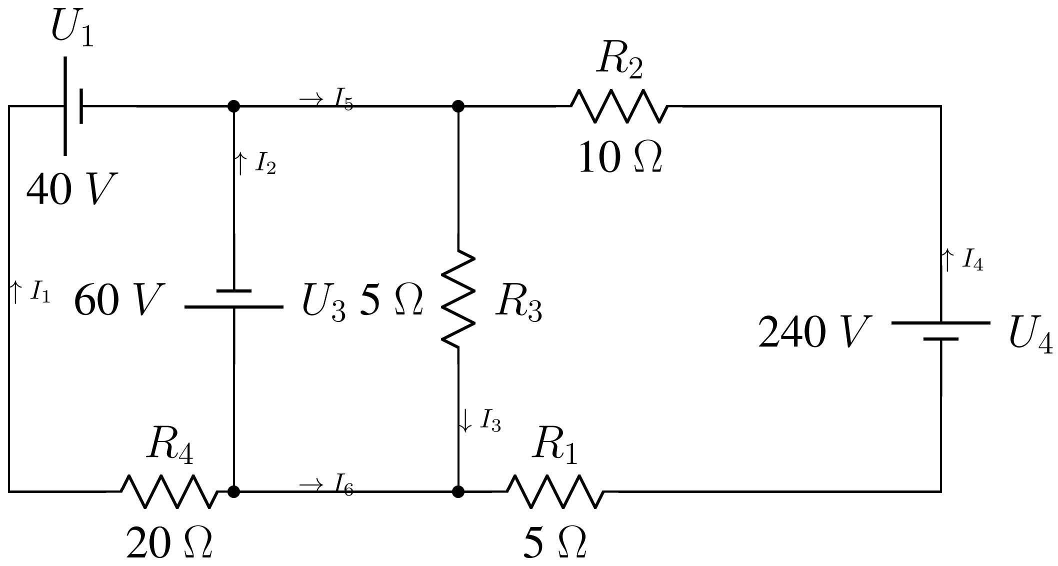
Good luck ! Bonne chance !

(1) Consider the electrical network below.



Recall that source of voltage | | has the plus pole on the left and the minus pole on the right. So if you start at the left side, then the voltage has positive sign (voltage decrease), otherwise negative sign (voltage increase).

(a) (4 pts) If $I_1, I_2, \ldots, I_6$ denote the electric currents in amperes, directed as indicated, apply the junction rule to obtain a homogeneous system of linear equations describing the currents when resistors and sources of voltage are ignored. Solve the system.

(b) (3 pts) For every closed circuit in the network choose a direction (clockwise or counterclockwise). Indicate this direction clearly in the diagram above. Using Ohm's law and the circuit rule to find the remaining equations needed to describe the current.

(c) (1 pt) The equations in (a) and (b) form a linear system in the variables $I_1, I_2, \ldots I_6$. Find the currents in the circuit by solving the system.

(2) Consider the following matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix},\ C = \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix},\ D = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}.$$

(a) (3 pts) Which of the following are defined ? **Justify your answer !** Do not compute the matrix product.

$$AB,\ BD,\ DB,\ AC,\ B^2,\ D^2.$$

(b) (2 pts) Calculate the following :

$$A^T B^T, \quad DD^T.$$

(c) (5 pts) Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix. Find necessary and sufficient conditions for a, b, c and d such that

$$XD = DX$$

where D is as above. Give an example of a matrix Y for which $YD \neq DY$, or explain why this is not possible.

(3) (8 pts) Determine whether each of the following vectors $V_1, \ldots, V_4$ is a linear combination of the vectors

$$X_1 = \begin{bmatrix}1 & 2 & 0\end{bmatrix}^T \quad \text{and} \quad X_2 = \begin{bmatrix}2 & 0 & -1\end{bmatrix}^T.$$

(a) $V_1 = \begin{bmatrix}2 & 0 & -1\end{bmatrix}^T$;

(b) $V_2 = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$;

(c) $V_3 = \begin{bmatrix}0 & 4 & 1\end{bmatrix}^T$;

(d) $V_4 = \begin{bmatrix}2 & 2 & -1\end{bmatrix}^T$.

(4) (4 pts) In the formula below replace α by the last digit of your student number and determine A :

$$\left(3A^T - \begin{bmatrix} 1 & \alpha & -2 \\ -4 & 5 & \alpha - 2 \end{bmatrix}\right)^T = \begin{bmatrix} -4 & 3 \\ 2 & \alpha + 3 \\ -2 & 6 \end{bmatrix} + 3 \begin{bmatrix} 7 & -5 & 4 \\ 9 & 12 & \alpha \end{bmatrix}^T .$$

(5) Let $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be the matrix transformations induced by the matrices M_1 and M_2 respectively, where

$$M_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad M_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.$$

(a) (4 pts) Find the general formula for the transformations T_1 and T_2, and determine the geometric meaning of T_1 and T_2.

(b) (3 pts) Find the matrix of the composition $T_3 = T_2 \circ T_1$, and determine the geometric meaning of the transformation T_3.

(6) (a) (3 pts) Suppose A is a 3×5-matrix of rank 3. Is the linear system $AX = B$ consistent for every $B \in \mathbb{R}^3$? If yes, explain why. If no, provide a concrete counter-example (concrete = with numbers).

(b) (3 pts) Suppose the homogeneous linear system $AX = 0$ has a nontrivial solution. If $AX = B$ is consistent, is it uniquely solvable ? Justify your answer !