

University of Ottawa
Department of Mathematics and Statistics

MAT 1341C : Introduction to Linear Algebra
Instructor : Erhard Neher

Assignment 1 : due Jan. 19, 2010, 13:00 in the classroom

FAMILY NAME (CAPITALS)	_____
FIRST NAME (CAPITALS)	_____
Signature	_____
Student number	_____

Please read these instructions carefully :

- The table below is for the TA. Do not write in it.
- The assignment has to be submitted with the two cover pages.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1	2	3	4	5	Total
Score						
Max. score	5	4	8	8	8	33

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FAMILY NAME (CAPITALS) _____

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Good luck ! Bonne chance !

- (1) (5 pts) Your doctor has asked you to take 5 units of vitamin A, 13 units of vitamin B and 23 units of vitamin C every day. There are three brands available in your local pharmacy which contain the following units of vitamins A, B, C as indicated.

	vitamin A	vitamin B	vitamin C
Brand 1	1	2	4
Brand 2	1	1	3
Brand 3	0	1	1

(a) (4 pts) Find all combination of pills that provide you with the exact daily requirement (no partial pills!).

(b) (1 pt) If brands 1, 2, and 3 cost \$0.90, \$0.60, and \$1.50 per pill respectively, find the least expensive treatment and its cost.

- (2) (4 pts) For each of the four matrices below determine if the matrix is in row-echelon form, in reduced row-echelon form or not in row-echelon form at all. Justify your answer.

(a)
$$\begin{bmatrix} 1 & 8 & 8 \\ 0 & 1 & 5 \\ 0 & 0 & -4 \end{bmatrix}$$

My answer: _____

(b)
$$\begin{bmatrix} 1 & 15 & 19 & 5 \\ 0 & 1 & 8 & 14 \\ 0 & 0 & -8 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

My answer: _____

(c)
$$\begin{bmatrix} 1 & 4 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

My answer: _____

(d)
$$\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

My answer: _____

- (3) (8 pts) In the matrices below the first entry in every row (■) is a non-zero number, while (*) represents an arbitrary number, which could be but need not be 0. Suppose that each matrix is the augmented matrix of a linear system. Determine in each case if the corresponding linear system is solvable (= consistent) or not. If the system is solvable, also determine if it is uniquely solvable, and if not determine the number of free parameters in the general solution. Justify your answer !!

(a)
$$\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & 0 \end{bmatrix}$$

My answer: _____

(b)
$$\begin{bmatrix} \blacksquare & * & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & * \end{bmatrix}$$

My answer: _____

(c)
$$\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$$

My answer: _____

(d)
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

My answer: _____

(4) (8 pts) True or false ? Justify your answer !

(a) The positions of the leading 1's in a row-echelon form of a matrix depend on how and in which order the rows have been multiplied during the application of the Gaussian algorithm.

My answer:_____

(b) If a linear system has free variables (variables which do not correspond to a leading 1 in a row-echelon form of the augmented matrix), the system has infinitely many solutions.

My answer:_____

(c) The general solution of a linear system is the explicit description of all solutions of the system.

My answer:_____

(d) We consider a linear system of 3 equations with 3 variables. If the reduced row-echelon form of the coefficient matrix of the system has a leading 1 in each column, the system is solvable.

My answer:_____

(5) (6+2 pts) Consider the following system of linear equations :

$$\begin{array}{rccccrcr} x & + & & y & - & z & = & 3 \\ -x & & & & & + & az & = & -4 \\ 3x & - & (a-5)y & - & 9z & = & 11 \end{array}$$

(a) Determine the values of a for which the system has :

- (i) no solution
- (ii) infinitely many solutions
- (iii) exactly one solution.

(b) In case (ii), describe the solution set of the system.