

# University of Ottawa

## Department of Mathematics and Statistics

**MAT 1341C: Introduction to Linear Algebra**

**Instructor: Erhard Neher**

**Diagnostic Test Jan. 17, 2009**

**Family Name:** \_\_\_\_\_

**First Name:** \_\_\_\_\_

**Student number:** \_\_\_\_\_

**Please read these instructions carefully:**

- Enter your name on this page and the next, but your student number only on this page. You will get back the exam without this first page.
- You have 80 minutes to complete this test.
- This is a closed book exam, and no notes of any kind are allowed. The use of calculators, cell phones, pagers or any text storage or communication device is not permitted.
- Read each question carefully - you will save yourself time and unnecessary grief later on.
- All 12 questions are multiple choice, are worth 1 point each and no part marks will be given. Please record your answers in the space provided.
- Where it is possible to check your work, do so.

**Good luck! Bonne chance!**

**University of Ottawa**

**Department of Mathematics and Statistics**

**MAT 1341C: Introduction to Linear Algebra**

**Instructor: Erhard Neher**

**Diagnostic Test Jan. 17, 2009**

**Family Name:** \_\_\_\_\_

**First Name:** \_\_\_\_\_

1. Find the scalar equations of the line passing through  $P(2, -5, 7)$  and parallel to the line through  $Q(-1, 1, -1)$  and  $R(1, -1, 1)$ .

- A.  $x = 9 + 2t, y = -7 - 2t, z = 4 + 2t, t$  scalar.
- B.  $x = -2 + 2t, y = -5 - 2t, z = 5 + 2t, t$  scalar.
- C.  $x = 2 + 3t, y = -3 - 2t, z = 7 + 2t, t$  scalar.
- D.  $x = 2 + 2t, y = -5 - 2t, z = 7 + 2t, t$  scalar.
- E.  $x = 2 + 2t, y = -5 - 2t, z = 7 - 2t, t$  scalar.
- F.  $x = 2 + t, y = -5 + 9t, z = 1 + t, t$  scalar.

My answer: \_\_\_\_\_

2. Find the direction vector of the line passing through  $P(3, -3, 3)$ , intersecting the line  $[x \ y \ z]^T = [9 \ -5 \ 7]^T + t[-1 \ 2 \ -1]^T$  and perpendicular to it.

- A.  $[-1 \ 2 \ -1]^T$
- B.  $[9 \ -5 \ 7]^T$
- C. Such a line does not exist.
- D.  $[0 \ -4 \ -9]^T$
- E.  $[-11 \ -8 \ -5]^T$
- F.  $[1 \ 4 \ 7]^T$ .

My answer: \_\_\_\_\_

3. Find the scalar equation of the line passing through  $P(3, -9, 0)$  and  $Q(-1, 2, 7)$ .

- A.  $x = 1 + 2t, y = -9 + t, z = 2t, t$  scalar.
- B.  $x = 3 - 4t, y = -9 + 11t, z = 7t, t$  scalar.
- C.  $x = 1 - 3t, y = 4 + 11t, z = 1 + 2t, t$  scalar.
- D.  $x = 1 + 2t, y = 1 + 4t, z = 2t, t$  scalar.
- E.  $x = 3 + 4t, y = -4 + t, z = 1 + 7t, t$  scalar.
- F.  $x = -3 + 4t, y = 9 + 11t, z = 7t, t$  scalar.

My answer: \_\_\_\_\_

4. Find the equation of the plane passing through  $A(1, 0, -7)$  and perpendicular to the line  $[x \ y \ z]^T = [5 \ 0 \ 5]^T + t[2 \ -1 \ 7]^T$ .

- A.  $2x + 3y + 6z = -1$
- B.  $x - 2y + 4z = 2$
- C.  $2x - y + 7z = -47$
- D.  $4x - y + 2z = -4$
- E.  $2x + y + z = -41$
- F.  $6x - y + 2z = -40$

My answer: \_\_\_\_\_

5. Find the equation of the plane containing the point  $P(1, 2, -3)$  and also containing the line through  $Q(2, 0, 1)$  with direction vector  $[1 \ 1 \ 1]^T$ .

A.  $2x - y - z = 3$

B.  $2x - y - z = 0$

C.  $x + y + z = 3$

D.  $x + y + z = 0$

E.  $6x + 3y + z = 9$

F.  $6x + 3y + z = 0$

My answer: \_\_\_\_\_

6. What is the distance between the point  $P(1, 1, 1)$  and the plane  $2x - 2y + z = 0$ ?

A.  $\frac{1}{3}$

B.  $\frac{1}{9}$

C.  $\frac{1}{\sqrt{3}}$

D. 9

E. 3

F.  $\sqrt{3}$ .

My answer: \_\_\_\_\_

7. Let  $u = [1 \ 1 \ 1]^T$  and  $v = [2 \ -2 \ 1]^T$ . Suppose  $w$  and  $x$  are vectors satisfying the following three conditions:

- $u = w + x$ ,
- $w$  is a scalar multiple of  $v$ , i.e.,  $w = sv$  for some scalar  $s \in \mathbb{R}$ ,
- $x$  is orthogonal to  $v$ .

Then  $x$  is

- A.  $\frac{1}{9}(7, 11, 8)$
- B.  $-\frac{1}{3}(7, 11, 8)$
- C.  $\frac{1}{9}(1, -1, -4)$
- D.  $-\frac{1}{3}(1, -1, -4)$
- E.  $\frac{1}{9}(8, 10, 4)$
- F.  $-\frac{1}{3}(8, 10, 4)$ .

My answer: \_\_\_\_\_

8. Find the area of the triangle with vertices  $A(2, 0, 1)$ ,  $B(0, 1, 2)$  and  $C(1, 1, -1)$ .

- A.  $\frac{1}{2}\sqrt{35}$
- B.  $\sqrt{35}$
- C. 25
- D. 25/2
- E. 5/2
- F.  $\frac{1}{2}\sqrt{5}$

My answer: \_\_\_\_\_

9. Find the distance from the point  $P(1, 0, 1)$  to the line through  $P_0(0, 0, 0)$  with direction vector  $d = [1 \ 1 \ 1]^T$ .

A.  $\frac{2}{3}$

B.  $2\sqrt{5}$

C. 0

D.  $\frac{1}{4}$

E.  $\frac{1}{7}\sqrt{5}$

F.  $\frac{1}{3}\sqrt{6}$

My answer: \_\_\_\_\_

10. Write

$$\frac{1+i}{2-i} - \frac{3-4i}{1+2i}$$

in the form  $a + ib$  with  $a, b \in \mathbb{R}$ .

A.  $\frac{1}{4}(7 - 8i)$

B.  $-\frac{1}{2}(9 - 5i)$

C.  $\frac{3}{7}(2 - 8i)$

D.  $\frac{4}{5}(6 + i)$

E.  $\frac{1}{5}(6 + 13i)$

F.  $-\frac{2}{3}(5 - 11i)$

My answer: \_\_\_\_\_

11. The roots of  $3x^2 + 2x + 1$  are

- A.  $\frac{3}{4}(1 \pm 2i)$
- B.  $\frac{1}{3}(-1 \pm \sqrt{2}i)$
- C. This polynomial does not have complex roots
- D.  $\frac{1}{6}(-4 \pm \sqrt{11}i)$
- E.  $\frac{2}{3}(1 \pm \sqrt{5})$
- F.  $\frac{1}{7}(2 \pm \sqrt{5}i)$

My answer: \_\_\_\_\_

12. Find the correct combination of true/false for the following three statements.

- For a complex number  $z$  and its complex conjugate  $\bar{z}$  we always have  $z\bar{z} \geq 0$ .
- The equation  $x^3 - 7x^2 + 43x - 13 = 0$  has a solution in the complex numbers.
- For a complex number  $z$  and a real number  $r$  we always have  $|rz| = |r||z|$ .

- A. true/true/true
- B. false/true/true
- C. true/false/true
- D. true/true/false
- D. false/false/true
- E. false/true/false
- F. false/false /false

My answer: \_\_\_\_\_