University of Ottawa
Department of Mathematics and Statistics

MAT 1341C: Introduction to Linear Algebra
Instructor: Erhard Neher

Test 2

Family name (CAPITALS)	________________
First name (CAPITALS)	________________
Signature	________________
Student number	________________

Please read these instructions carefully:

- The table below is for the TA. Do not write in it.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1	2	3	4	5	6	7	8	Total
Score									
Maximal score	3	3	3	3	5	5	5	2 bonus	27

University of Ottawa
Department of Mathematics and Statistics

MAT 1341C: Introduction to Linear Algebra

Test 2

FAMILY NAME (CAPITALS) ______________________________

FIRST NAME (CAPITALS) ______________________________

Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 5 – 7 you may use the back of pages if necessary, but be sure to indicate to the marker that you have done so.
- No part marks will be given for questions 1 – 4. However, you must show some work to obtain the point. Simply writing the correct answer will earn you 0.
- Question 8 is a bonus proof question. You can get 2 extra points.
- No books or notes are allowed. **Calculators are not permitted.**

Good luck! Bonne chance !

(1) (3 pts) Let U be a subspace of $\mathbb{R}^7$ which is spanned by 6 vectors. Are the following claims true or false? Answer with T for "true" or F for "false".

(a) $\dim U \leq 6$.

My answer:________________

(b) Any set of $\dim U$ vectors in U is linearly independent.

My answer:________________

(c) Every spanning set of U has at most 6 vectors.

My answer:________________

(2) (3 pts) Let A be a 4×9 matrix. Are the following claims true or false? Answer with T for "true" or F for "false".

(a) If A has rank 4, then the columns of A^T are linearly independent.

My answer:________________

(b) If A has rank 3, the null space of A is a subspace of $\mathbb{R}^6$.

My answer:________________

(c) If A has rank 4, then every set of 4 columns of A is linearly independent.

My answer:________________

(3) (3 pts) (a) The dimension of the vector space $\mathbb{M}_{34}$ of 3×4-matrices is

My answer:__________________

(b) The dimension of the vector space $\mathbb{P}_5$ of polynomials of degree ≤ 5 is

My answer:__________________

(c) Give a basis of $\mathbb{M}_{22}$ consisting of non-invertible matrices:

(4) (3 pts) Are the following subsets U subspaces of the indicated vector spaces V? Answer with Y for "yes" or N for "no".

(a) $U = \{f \in V : f(0) = 0 = f(1)\}$ in $V = \mathbb{F}[0, 3]$.

My answer:__________________

(b) $U = \{A \in V : A^T = A\}$ in $V = \mathbb{M}_{2,2}$.

My answer:__________________

(c) $U = \{p \in V : p(0) = 0, p(1) = 1\}$ in $V = \mathbb{P}_3$.

My answer:__________________

(5) (5 pts) Find all $x \in \mathbb{R}$ such that $\begin{bmatrix} x+7 & 2 & -3 \\ -4 & x+1 & 3 \\ -3 & 3 & x+1 \end{bmatrix}$ is NOT invertible.

(6) (5 pts) Consider the subspace U of $\mathbb{R}^4$ spanned by $\{v_1, v_2, v_3, v_4, v_5\}$ where:

$$v_1 = \begin{bmatrix} 1 \\ -4 \\ 7 \\ -4 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -2 \\ -4 \\ 5 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ -5 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 5 \\ 4 \\ -3 \\ -6 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 3 \\ -2 \\ -7 \\ 6 \end{bmatrix}.$$

Find a basis of U and give the dimension of U.

(7) (5 pts) Show that

$$U = \{p \in \mathbb{P}_2 : p(2) = 0\}$$

is a subspace of $\mathbb{P}_2$, find a basis of U and determine its dimension.

My answer for basis:________________________________

My answer for the dimension:________________________________

(8) (2 bonus points) (a) Give the definition of a basis of an arbitrary vector space V.

(b) Prove that the vector space $\mathbb{P}$ of all polynomials does not have a basis.