University of Ottawa
Department of Mathematics and Statistics

MAT 1341C: Introduction to Linear Algebra
Instructor: Erhard Neher

Test 1

Family name (CAPITALS)	____________________
First name (CAPITALS)	____________________
Signature	____________________
Student number	____________________

Please read these instructions carefully:

- The table below is for the TA. Do not write in it.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages. Therefore, **fill in your name on both pages and your student number on this page only.**

Question	1.	2	3	4	5	6	7	Total
Score								
Max. score	5	3	2	2	8	8	2 bonus	28

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FAMILY NAME (CAPITALS)	______________________
FIRST NAME (CAPITALS)	______________________

Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 5 and 6 you may use the back of pages if necessary, but be sure to indicate to the marker that you have done so.
- No part marks will be given for questions 1 – 4. However, you must show some work to obtain the point. Simply writing the correct answer will earn you 0.
- Question 7 is a bonus proof question. You can get 2 extra points.
- No books or notes are allowed. **Calculators are not permitted.**

Good luck! Bonne chance !

(1) (5 pts) In each case give an example of:

(a) (1 pts) An inconsistent linear system of 2 equations in 2 variables.

(b) (2 pts) A linear system of 3 equations in 2 variables which has infinitely many solutions. Also, give at least two different solutions.

(c) (2 pts) An example of a linear system which is uniquely solvable. Also, give the unique solution.

(2) (3 pts) Complete the theorem below by stating 3 conditions which are equivalent to, but not the same as the condition in (a).

Theorem. *For a $n \times n$ matrix A the following conditions are equivalent:*

(a) *A is invertible.*

(b)

(c)

(d)

Remarque : The theorem stated in class had more equivalent conditions. But you are only asked to list 3 of them.

(3) (2 pts) Consider the following matrices:

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & -5 & 0 & 3 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & 1 & 2 & -3 & 5 \\ 0 & 0 & 3 & 0 & -2 \\ 1 & 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & -1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & -6 \end{bmatrix}.$$

Which one is or which ones are in reduced row-echelon form?

My answer:__________________

(4) (2 pts) Let $A = \begin{bmatrix} 2 & 7 & 1 \\ 5 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ et $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. Calculate the matrix $B(AB)^{-1}A^2$.

My answer:__________________

(5) For the system of linear equations

$$\begin{array}{ccccccc} x & & & + & z & = & 1 \\ 3x & + & (a-3)y & + & az & = & 3 \\ 9x & & & + & a^2 z & = & a+6 \end{array}$$

(a) (6 pts) determine the values of a for which the system has

(i) no solution,

(ii) infinitely many solutions,

(iii) a unique solution.

(b) (2 pts) In case (ii) above describe give all solutions.

(6) (8 pts) In the matrix below **replace α with the second-last digit of your student number** and find its inverse:

$$A = \begin{bmatrix} 1 & \alpha & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 5 \end{bmatrix}$$

Check your answer by verifying $AA^{-1} = I_3$.

(7) (2 bonus points) (a) Give the definition for a $n \times n$ matrix A to be invertible.

(b) Show that the inverse of an invertible matrix is unique.