

# Page 1

University of Ottawa

Department of Mathematics and Statistics

MAT 1341: Introduction to Linear Algebra

Instructor: Erhard Neher

Assignment 4; due July 9, 2008, 18:00 in the class room

Family Name: \_\_\_\_\_

First Name: \_\_\_\_\_

Student number: \_\_\_\_\_

Please read these instructions carefully:

- The table below is for the TA. Do not write in the table.
- The assignment has to be submitted with the two cover pages. You may or may not use the pages 3-7 of this copy.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the assignment. Therefore, **fill in your name on both pages and your student number on this page only.**

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Quest.	1.	2.	3.	4.	5.	Total
maximal	5	9	6	7	4 extra	27
score						

## Page 2

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Family Name: \_\_\_\_\_

First Name: \_\_\_\_\_

**Please read these instructions carefully:**

- All questions require justification, written legibly and logically: You must convince the TA and me that you know why your solution is correct. Correct answers without justification will get 0 marks.
- You have to submit this assignment at the beginning of the class on Wednesday, July 9, 2008, at 18:00 in the classroom at the latest. If you wish to submit it earlier, please do so at the secretariat of the Department of Mathematics, room 103A, 8:45–12:00 and 13:00–16:00.

**Good luck! Bonne Chance!**

1. (5 points) Determine if

$$\left\{ \frac{1}{x^2+x-6}, \frac{1}{x^2-x-2}, \frac{1}{x^2+4x+3} \right\}$$

is a linearly independent subset of the vector space  $V = \mathbb{F}[0, 1]$ . If not, give a nontrivial linear combination which vanishes.

- 2.** (a) **(5 points)** Show that  $U = \{A \in \mathbb{M}_{33} : A^T = -A\}$  is a subspace of the vector space  $V = \mathbb{M}_{33}$ , find a basis of  $U$  and determine its dimension.
- (b) **(4 points)** Find a linear map  $T : \mathbb{M}_{33} \rightarrow \mathbb{M}_{33}$  such that  $U = \ker(T)$ . Use this to give a second proof that  $U$  is a subspace.

**3. (6 points)** A polynomial  $p(x)$  is called *odd* if  $p(x) = -p(-x)$ . Show that the set  $U$  of odd polynomials in  $\mathbb{P}_n$  is a subspace, find a basis of  $U$  and determine its dimension.

4. (a) (4 points) Let  $U$  be a nonzero subspace of  $\mathbb{R}^n$ . Show that  $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ ,  $T(X) = \text{proj}_U(X)$  is a linear transformation and determine  $\ker(T)$  and  $\text{im}(T)$ .

(b) (3 points) For  $U = \text{Span}\{[1 \ -1 \ 1 \ 2]^T, [1 \ 1 \ 2 \ -1]^T\} \subset \mathbb{R}^4$  determine the standard matrix of the linear transformation  $T$  of (a).

**5. (4 extra points)** Let  $V$  and  $W$  be vector spaces and let  $T : V \rightarrow W$  be a linear map.

(a) Let  $Z \subset W$  be a subspace of  $W$ . Show that  $U = \{v \in V : T(v) \in Z\}$  is a subspace of  $V$ .

(b) Let  $U_1 \subset V$  be a subspace of  $V$ . Show that  $Z_1 = \{w \in W : w = T(u_1) \text{ for some } u_1 \in U_1\}$  is a subspace of  $W$ .