University of Ottawa

Department of Mathematics and Statistics

MAT 1341: Introduction to Linear Algebra
Instructor: Erhard Neher

Assignment 3; due June 25, 2008, 18:00 in the class room

Family Name: ______________________________

First Name: ______________________________

Student number: ___________________________

Please read these instructions carefully:

- The table below is for the TA. Do not write in the table.
- The assignment has to be submitted with the two cover pages. You may or may not use the pages 3-7 of this copy.
- For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the assignment. Therefore, **fill in your name on both pages and your student number on this page only.**

Quest.	1.	2.	3.	4.	5.	Total
maximal	5	5	5	5	3 extra	20
score						

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Please read these instructions carefully:

- All questions require justification, written legibly and logically: You must convince the TA and me that you know why your solution is correct. Correct answers without justification will get 0 marks.
- You have to submit this assignment at the beginning of the class on Wednesday, June 25, 2008, at 18:00 in the classroom at the latest. If you wish to submit it earlier, please do so at the secretariat of the Department of Mathematics, room 103A, 8:45–12:00 and 13:00–16:00.

Good luck! Bonne Chance!

1. (5 points) Find a basis of the row and column space of A and determine the rank of A, where A is the matrix

$$\begin{bmatrix} 1 & -2 & 3 & 0 & 0 \\ 2 & -5 & 6 & -3 & -2 \\ 0 & 5 & 0 & 15 & 10 \\ 2 & 6 & 6 & 18 & 8 \end{bmatrix}$$

2. (5 points) Use the Gram-Schmidt algorithm to convert the basis

$$[1 \quad -1 \quad 1]^T, \quad [1 \quad 0 \quad 1]^T, \quad [1 \quad 1 \quad 2]^T$$

of $\mathbb{R}^3$ to an orthogonal basis.

3. (5 points) Write the vector X as a sum of a vector in U and in $U^{\perp}$:

$$X = [1 \quad 1 \quad 1]^T, \quad U = \text{Span}\,\{[1 \quad -1 \quad 2]^T, [3 \quad -1 \quad 4]^T\}.$$

4. (5 points) Find the best approximation to a solution of the following linear system:

$$\begin{array}{rcrcr} x & - & y & = & 3 \\ 2x & + & y & = & -1 \\ x & + & 5y & = & -4 \end{array}$$

Also determine the vector among all vectors AX which is closest to $B = [\,3 \quad -1 \quad -4\,]^T$.

5. (3 extra points) (Continuation of question 5 of Assignment 2) Let U and V be subspaces of $\mathbb{R}^n$. Recall that then $U \cap V = \{x \in \mathbb{R}^n : x \in U \text{ and } x \in V\}$ and $U + V = \{x \in R^n : x = u + v \text{ for some } u \in U \text{ and } v \in V\}$ are subspaces of $\mathbb{R}^n$. Show that

$$\dim U + \dim V = \dim(U + V) + \dim(U \cap V).$$

(Hint: If $U \cap V \neq \{0\}$ let B be a basis of $U \cap V$, otherwise put $B = \emptyset$. By Theorem 3 of §4.3, one can extend B to a basis B_U of U and also to a basis B_V of V. Show that $B_U \cup B_V$ is a basis of $U + V$.)