University of Ottawa

Department of Mathematics and Statistics

MAT 1341: Introduction to Linear Algebra
Instructor: Erhard Neher

Test 2; June 30, 2008, 17:00-18:15 in the class room

Family Name: ______________________________

First Name: ______________________________

Student number: ______________________________

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. You will get back the exam without this first page.
- The table below is for the TA. Do not write in the table. For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the test. Therefore, **fill in your name on both pages** and your student number on this page only.
- No books or notes are allowed. **Calculators are not permitted.**

Quest.	$1-2$	$3-4$	5	6	7	8	Total
maximal	6	6	5	5	5	2 bonus	27
score							

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 5, 6 and 7 you may use the back of pages if necessary, but be sure to indicate to the marker that you have done so.
- Question 8 is a bonus proof question. You can get 2 extra points.
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Good luck! Bonne Chance!

1. (3 points) Write $X = [1 \quad 3 \quad 1 \quad 0]^T$ as a linear combination of the vectors $[2 \quad 4 \quad 0 \quad -6]^T$ and $[0 \quad 3 \quad 3 \quad 9]^T$.

My answer:____

2. Let U be a subspace of $\mathbb{R}^{10}$ which is spanned by 6 vectors. Are the following claims true or false? Answer with T for "true" or F for "false".

(a) (1 point) $\dim U = 6$.

My answer:______________

(b) (1 point) Any set of 7 vectors in U is linearly dependent.

My answer:______________

(c) (1 point) Every spanning set of U has at most 6 vectors.

My answer:______________

3. Let A be a 3×7 matrix. Are the following claims true or false? Answer with T for "true" or F for "false".

(a) (1 point) If the rows of A are linearly independent, then $rowA = \mathbb{R}^4$.

My answer:________________

(b) (1 point) The dimension of the null space of A is at least 4.

My answer:________________

(c) (1 point) If A has a row of zeros, then $\operatorname{rank}(A) \leq 2$.

My answer:________________

4. Are the following subsets U subspaces of the indicated vector spaces V? Answer with Y for "yes" or N for "no".

(a) (1 point) $U = \{xp(x) : p \in \mathbb{P}_3\}$ in $V = \mathbb{P}$.

My answer:________________

(b) (1 point) $U = \{A \in \mathbb{M}_{2,2} : \det(A) = 1\}$ in $V = \mathbb{M}_{2,2}$.

My answer:________________

(c) (1 point) $U = \{f \in \mathbb{F}[0,1] : f(\frac{1}{2}) = 0\}$ in $V = \mathbb{F}[0,1]$.

My answer:________________

5. (5 points) Show that

$$U = \{ [\,a \quad b \quad c\,]^T \in \mathbb{R}^3 : a - b + c = 0 \}$$

is a subspace of $\mathbb{R}^3$, find a basis of U and calculate its dimension.

My answer for basis:______________________________

My answer for the dimension:______________________________

6. (5 points) Find a basis of the row space of A and of the column space of A, and determine the rank of A where A is the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 & 0 \\ 4 & 8 & -3 & 10 & 4 \\ 2 & 4 & 1 & 3 & 15 \\ -1 & -2 & 0 & -1 & -4 \end{bmatrix}.$$

My answer for a basis of the row space of A:____________________________

My answer for a basis of the column space of A: ____________________________

My answer for the rank of A:____________________________

7. (a) **(3 points)** Apply the Gram-Schmidt algorithm to convert

$$X_1 = [1 \quad 1 \quad 1 \quad 1]^T, \quad X_2 = [3 \quad 1 \quad 3 \quad 1]^T, \quad X_3 = [1 \quad 3 \quad -1 \quad 1]^T$$

into an orthogonal basis of $U = \text{Span}\{X_1, X_2, X_3\}$. (Continued on next page.)

(b) **(2 points)** The vectors $F_1 = [\,1 \quad -2 \quad 1\,]^T$ and $F_2 = [\,0 \quad 1 \quad 2\,]^T$ are orthogonal. Find the orthogonal projection of the vector $X = [\,11 \quad 4 \quad 3\,]^T$ onto the subspace $U = \text{Span}\,\{F_1, F_2\}$.

8. (2 bonus points) Let $S = \{X_1, X_2, \ldots, X_k\}$ be an orthogonal subset of $\mathbb{R}^n$. Prove that S is linearly independent.