University of Ottawa

Department of Mathematics and Statistics

MAT 1341: Introduction to Linear Algebra
Instructor: Erhard Neher

Test 1; June 2, 2008, 17:00-18:15 in the class room

Family Name: ______________________________

First Name: ______________________________

Student number: ___________________________

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. You will get back the exam without this first page.
- The table below is for the TA. Do not write in the table. For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the test. Therefore, **fill in your name on both pages** and your student number on this page only.
- No books or notes are allowed. **Calculators are not permitted.**

Quest.	1 – 2	3 – 4	5	6	7	8	Total
maximal	6	6	4	8	8	2 bonus	32
score							

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 6 and 7 you may use the back of pages if necessary, but be sure to indicate to the marker that you have done so.
- Question 8 is a bonus proof question. You can get 2 extra points.
- No books or notes are allowed. **Calculators are not permitted.**

Good luck! Bonne Chance!

1. (3 points) Answer the following questions.

(a) Suppose a row-echelon form of a 5×7 matrix A has exactly 3 leading 1's. If $AX = B$ is consistent, the general solution will depend on how many parameters?

My answer:________________

(b) (1 point) A homogeneous linear system has 4 equations and 5 variables. Does it have infinitely many solutions?

My answer:________________

(c) (1 point) A consistent linear system $AX = B$ has 4 equations and 5 variables. How many parameters are possible in the general solution if one row of the augmented matrix of $AX = B$ is a multiple of another row and $A \neq 0$?

My answer:________________

2. (3 points) Let A be an $n \times n$-matrix. Give 3 conditions that are equivalent but different from the condition

"The homogeneous linear system $AX = 0$ has a nontrivial solution."

(I)

(II)

(III)

3. (3 points) Let A and B be $n \times n$ matrices. True or false?

(a) If $A \neq 0$ then A is invertible.

My answer:________________

(b) If $A^2 = 2I_n$ then A is invertible.

My answer:________________

(c) If A and B are invertible, then AB is invertible and its inverse is $(AB)^{-1} = A^{-1}\,B^{-1}$.

My answer:________________

4. (a) (1 point) Draw the directed graph whose adjacency matrix is

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(number the vertices $1, 2, 3, 4$)

(b) (2 points) In this graph there are no paths of length 2 from any vertex to the vertex 2. How can this be explained in terms of matrix multiplication?

5. (4 points) Find A if

$$\left(2A^T + \begin{bmatrix} 14-i & 0 \\ 0 & -3 \end{bmatrix}\right)^{-1} = \begin{bmatrix} -i & 8i \\ 2i & 1-16i \end{bmatrix}.$$

6. (8 points) Consider the system of linear equations

$$\begin{array}{ccccc} (a+5)x & + & 12y & = & 4 \\ x & + & (2-a)y & = & 1 \end{array}$$

where a is a real parameter. Find the conditions on a, so that the system has

(i) no solution,

(ii) infinitely many solutions, and

(iii) a unique solution.

In case (ii) write down all solutions.
(Distribution of points: row reduction 3 points, 1 point for each of the 3 cases, solutions in (ii): 2 points.

7. (8 points) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 6 & 3 & 7 \\ 3 & 2 & 4 \end{bmatrix}.$$

Check your answer by calculating $AA^{-1} = I_3$. (Hint: A^{-1} is a matrix in which all entries are integers.)

8. (**2 bonus points**) Let A be an $m \times n$-matrix and B an $n \times p$-matrix. Prove that $(AB)^T = B^T A^T$.