University of Ottawa

Department of Mathematics and Statistics

MAT 1341: Introduction to Linear Algebra
Instructor: Erhard Neher

Test 2; June 26, 2006, 17:00-18:15 in the class room

Family Name: ______________________________

First Name: ______________________________

Student number: ___________________________

Please read these instructions carefully:

- Enter your name on this and the next page, but your student number only on this page. You will get back the exam without this first page.
- The table below is for the TA. Do not write in the table. For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the test. Therefore, **fill in your name on both pages** and your student number on this page only.
- No books or notes are allowed. **Calculators are not permitted.**

Quest.	$1-6$	7	8	9	Total
maximal	6	6	6	2 (bonus)	18
score					

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 7 and 8, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
- Questions 1 to 6 are worth 1 point each, and no part marks will be given. However, you must show some work to obtain the point. Simply writing the correct answer will earn you 0.
- The correct answers to questions 7-9 require justification written legibly and logically: You must convince the TA and me that you know why your solution is correct.
- No books or notes are allowed. **Calculators are not permitted.**

Good luck! Bonne Chance!

1. (1 point) If $\det(A) = -\frac{1}{2}$, $\det(B) = 4$ and $\det(C) = 5$ then $\det(A^4\, B^2\, C^T\, A^{-1})$ is

A. -20.

B. $-\frac{2}{5}$.

C. -10.

D. 20.

E. $-\frac{5}{2}$.

F. 4.

My answer:________________

2. (1 point) The eigenvalues of the matrix $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ are

A. 4 and -1.

B. this matrix does not have eigenvalues, not even complex ones.

C. $\pm\sqrt{3}$.

D. only 1 is an eigenvalue.

E. $1 \pm i$.

F. $2 \pm i\sqrt{3}$.

My answer:________________

3. (1 point) Find all values of x for which the vectors $(1, 2, 2)$, $(0, 1, 3-x)$ and $(1, x, 0)$ are linearly independent.

A. $x \neq -2$ and $x \neq -6$.

B. $x \neq 1$ and $x \neq 4$.

C. $x \neq 0$ and $x \neq -3$.

D. $x = 2$ and $x = -4$.

E. $x \neq 3$ and $x \neq 5$.

F. $x \neq 2$ and $x \neq 5$.

My answer:________________

4. (1 point) If the vector $(0, -5, 4, -9)$ is written as a linear combination of the vectors

$$(1, 0, 0, 1), \quad (0, 1, 4, 1) \quad \text{and} \quad (1, 3, 0, 6)$$

then the sum of the coefficients in this linear combination is

A. 1.

B. 2.

C. 3.

D. 0.

E. 4.

F. -2.

My answer:________________

5. (1 point) Let U be a subspace of $\mathbb{R}^n$ with $\dim U = p$. Which of the following statements are true? Give the correct combination of answers.

- Any set of $p+1$ vectors in U is linearly dependent.
- Any set of $p+1$ vectors in U is a spanning set of U.
- Any set of p linearly independent vectors in U is a basis of U.

A. true, true, true

B. false, true, true

C. true, false, true

D. true, true, false

E. true, false, false

F. false, true, false

My answer:________________

6. (1 point) Which of the following are subspaces of the indicated vector spaces?

$$U = \{f \in \mathbb{F}[0,2] : f(1) = 1\}, \quad V = \{p \in \mathbb{P}_2 : p(1) = 0\}, \quad W = \{A \in \mathbb{M}_{2,2} : A^2 = A\}.$$

A. U and W only.

B. V and W only.

C. U and V only.

D. Only U.

E. Only V.

F. Only W.

My answer:________________

7. (6 points) For the matrix

$$A = \begin{bmatrix} 2 & -2 & 1 & 3 \\ 1 & -1 & 1 & 2 \\ 5 & -5 & 1 & 6 \end{bmatrix}$$

determine

(a) (2 points) the reduced row-echelon form of A,

(b) (1 point) a basis of the row space of A,

(c) (1 point) a basis of the column space of A,

(d) (2 points) a basis of the null space of A.

8. (6 points) Find a basis and the dimension of the subspace

$$U = \{[\,a \quad b \quad c \quad d\,] \in \mathbb{R}^4 : 2a - 4b = 2c \text{ and } 3a + 5b - d = 0\}$$

of $\mathbb{R}^4$. **Justify your answer**, i.e., you must either show that you have a basis or quote some theorems from class. You do not have to show that U is a subspace.

9. (2 bonus points) Let A be an invertible $n \times n$ matrix. Use the Product Theorem for determinants to show that $\det(A) \neq 0$ and

$$\det(A^{-1}) = \frac{1}{\det(A)}\,.$$