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University of Ottawa

Department of Mathematics and Statistics

MAT 1341: Introduction to Linear Algebra

Instructor: Erhard Neher

Test 1; May 29, 2006, 17:00-18:15 in the class room

Family Name: _____

First Name: _____

Student number: _____

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. You will get back the exam without this first page.
- The table below is for the TA. Do not write in the table. For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the test. Therefore, **fill in your name on both pages** and your student number on this page only.
- No books or notes are allowed. **Calculators are not permitted.**

Quest.	$1-6$	7	8	Total
maximal	6	6	8	20
score				

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. For questions 7 and 8, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
- Questions 1 to 6 are worth 1 point each, and no part marks will be given. However, you must show some work to obtain the point. Simply writing the correct answer will earn you 0.
- Question 7 is a short answer question, no partial marks will be given. Question 8 is worth 8 points, part marks can be earned. The correct answers here require justification written legibly and logically: You must convince the TA and me that you know why your solution is correct.
- No books or notes are allowed. **Calculators are not permitted.**

Good luck! Bonne Chance!

1. (1 point) The real part of

$$\frac{(2-3i)(4+i)}{(6-2i)(1+i)}$$

is

A. 3/4
B. 33/20
C. 3/5
D. 33/21
E. 20/33
F. 4/5

My answer:_____

2. (1 point) An equation of the plane passing through $A(3,-1,4)$, $B(-1,5,1)$ and $C(0,2,-2)$ is:

A. $4x-9y+36z=18$
B. $9x+5y-2z=14$
C. $7x-8y+5z=6$
D. $8x-11y+18z=24$
E. $3x-2y+z=0$
F. $3x+2y-z=0$

My answer:_____

3. (1 point) Among the matrices

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 1 & 2 & -3 & 5 \\ 0 & 0 & 3 & 0 & -2 \\ 1 & 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & -1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & -6 \end{bmatrix}.$$

the following are in row-echelon form, but not in reduced row-echelon form

A. A, C and E.
B. A, B and D.
C. A, B and C.
D. B and C only.
E. A and E only.
F. B and E only.

My answer:_____

4. (1 point) If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and B is a $2 \times n$ matrix then the second row of the matrix AB is

A. not defined unless $n = 2$.
B. the same as the second row of B.
C. the same as the second row of A.
D. the same as the first row of B.
E. the same as the first row of A.
F. the sum of the first and the second row of B.

My answer:_____

5. (1 point) Compute the determinant $\begin{vmatrix} 0 & 0 & 0 & 7 & 0 \\ 3 & 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{vmatrix}$.

A. -25

B. -21

C. 33

D. -41

E. -35

F. 45

My answer:_____

6. (1 point) Which combination of true/false is correct for the following statements for $n \times n$ matrices A and B:

- If $A \neq 0$ then A is invertible.
- If A is invertible then $A \neq 0$.
- If $AB = I_n$ then A is invertible and $B = A^{-1}$.

A. true, false, true

B. false, true, false

C. true, true, false

D. false, false, false

E. true, false, false

F. false, true, true

My answer:_____

7. (6 points) Suppose a row-echelon form of a 4×4 matrix A has 2 leading 1's. Answer the following questions with justification (no justification = no credit).

(a) (1 point) Is the system $AX = B$ consistent for any choice of vectors B in $\mathbb{R}^4$?

(b) (1 point) If $AX = B$ is consistent, will there be infinitely many solutions?

(c) (1 point) Does the system $AX = 0$ have a unique solution?

(d) (1 point) If B is a linear combination of the columns of A, is then the linear system $AX = B$ solvable?

(e) (1 point) Is A^T invertible?

(f) (1 point) Is $\det(A) \neq 0$?

8. (8 points) Consider the system of linear equations

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 0 \\ kx & + & 4y & + & z & = & 12 \\ -x & - & 2y & + & kz & = & 4 \end{array}$$

where k is a real parameter. Find the conditions on k, so that the system has

(i) infinitely many solutions,
(ii) a unique solution, and
(iii) no solution.

In case (i) write done all solutions.