University of Ottawa

Department of Mathematics and Statistics

MAT 1341: Introduction to Linear Algebra

Instructor: Erhard Neher

Assignment 2; due June 21, 2006, 18:00 in the class room

Family Name: ______________________________

First Name: ______________________________

Student number: ______________________________

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. You will get back the assignment without this first page.
- The table below is for the TA. Do not write in the table. For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the assignment. Therefore, **fill in your name on both pages** and your student number on this page only.

Quest.	1	2	3	4	5	6	7	Total
maximal	3	3	3	3	3	6	3	24
score								

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. You may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
- For all questions you must show your work to obtain the points. Simply writing the correct answer will earn you 0.
- Please write legibly and argue logically: You must convince the TA that you know why your solution is correct.
- You have to submit this assignment at the beginning of the class on Wednesday, June 21, at 18:00 in the classroom at the latest. If you wish to submit it earlier, please do so at the secretariat of the Department of Mathematics, room 103A, 8:45–12:00 and 13:00–17:00.

Good luck! Bonne Chance!

1. (3 points) Let $x_1, x_2, \ldots, x_n$ be real numbers, $n \geq 2$. Show that

$$\begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{vmatrix} = \prod_{1 \leq j < i \leq n} (x_i - x_j)$$

where $\prod_{1 \leq j < i \leq n} (x_i - x_j)$ means the product of all factors $(x_i - x_j)$ for all pairs (i, j) satisfying $j < i$ and i and j between 1 and n. Hint: Replace the i^{th}-column C_i of the matrix in question by $C_i - x_1 C_{i-1}$, and expand along the first row. Then reduce to the analogous determinant for $n - 1$ numbers.

2. (3 points) Solve the following linear system using Gaussian elimination

$$\begin{array}{rcrcl} (1+i)x & + & (2+i)y & = & 5 \\ (2-2i)x & + & iy & = & 1+2i \end{array}$$

3. (3 points) Find the solution of the following system of linear differential equations

$$\begin{array}{rcrcl} f_1 & + & 3f_2 & = & f_1', \qquad f_1(0) = -1, \\ 2f_1 & + & 2f_2 & = & f_2', \qquad f_2(0) = 8. \end{array}$$

4. (3 points) Are the following subspaces of $\mathbb{R}^3$? Provide a short justification!

(a) $U = \{(x, y, z) \in \mathbb{R}^3 : 2x - 3y = 0, y + 4z = 0\}$,

(b) $V = \{(x, y, z) \in \mathbb{R}^3 : x^2 = y^2\}$,

(c) $W = \left\{ \begin{bmatrix} 2s - t \\ 3s + t \\ 4t \end{bmatrix} : s, t \in \mathbb{R} \right\}$.

5. (3 points) In each case either show that the statement is true or give an example showing that it is false. Throughout X_1, X_2 are vectors in $\mathbb{R}^n$.

(a) If $\{X_1, X_2\}$ is linearly independent, then so is $\{X_1, X_2, X_1 - X_2\}$.

(b) If $\{X_1, X_2\}$ is linearly dependent, then so is $\{X_1, X_2, X_1 - X_2\}$.

(c) If $\{X_1, X_2\}$ is linearly independent, then $c_1X_1 + c_2X_2 = 0$ for some $c_1, c_2 \in \mathbb{R}$.

6. (a) **(3 points)** Diagonalize the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}.$$

(b) **(3 points)** Solve the recurrence relation

$$x_{k+2} = 3x_k + 2x_{k+1}, \qquad x_0 = 1 = x_1,$$

i.e., find a closed form expression for x_k.

7. Let A be an $n \times n$ matrix. (a) **(1 point)** If λ is an eigenvalue of A show that $3\lambda^2 - 3\lambda + 5$ is an eigenvalue of $3A^2 - 3A + 5I_n$ where I_n is the $n \times n$ Identity matrix.

(b) **(2 points)** Suppose $A^2 = 0 \neq A$. Show that $\lambda = 0$ is the only eigenvalue of A. (You have to show that (i) if λ is an eigenvalue of A then necessarily $\lambda = 0$, and (ii) $\lambda = 0$ is an eigenvalue of A)