University of Ottawa

Department of Mathematics and Statistics

MAT 1341 B: Introduction to Linear Algebra

Instructor: Erhard Neher

Test 3; Nov. 22, 2007, 17:30-18:50

Family Name: ______________________________

First Name: ______________________________

Student number: ______________________________

The last digit of your student number is $\alpha =$

The second last digit of your student number is $\beta =$

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. You will get back the test without this first page.
- The table below is for the TA. Do not write in the table. For privacy reasons, this page of the test will be detached, and you will only get back the remaining pages of the test. Therefore, **fill in your name on both pages** and your student number on this page only.
- No books or notes are allowed. **Calculators are not permitted.**

Good luck!

Quest.	1	2	3	4	5	Total
maximal	6	6	$3 + 3$ bonus	8	6	29
score						

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Family Name: ______________________________

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. You may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
- For all questions you must show your work to obtain the points. Simply writing the correct answer will earn you 0.
- Please write legibly and argue logically: You must convince the TA that you know why your solution is correct.
- No books or notes are allowed. **Calculators are not permitted.**

1. (6 points) Let α denote the LAST digit of your student number. For each of the following three sets of vectors, answer the three questions:
(a) Is the set linearly independent?
(b) Do the vectors span $\mathbb{R}^3$?
You must correctly justify your answer to obtain any marks. (This question has two pages.)
(i)

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ \alpha - 9 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$$

(ii)

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ \alpha - 9 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$$

(iii)

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} \right\}$$

2. (6 points) For each of the sets given below:
(a) Give an example of an element in the set.
(b) Is it a subspace of $\mathbb{M}_{2,2}$? Justify your answer.

(i)

$$U = \{A \in \mathbb{M}_{2,2} : A = A^T\}$$

(ii)

$$W = \{A : \det(A) = 1\}$$

3. (a) (3 points) Prove that the set

$$\{1 + x - x^2,\ 2 - x,\ 1 + x^3\}$$

is a linearly independent subset of $\mathbb{P}_3$.

(b) (**3 bonus points**) Prove that the set

$$\{\frac{1}{x-2}, \frac{1}{x-3}, \frac{1}{(x-2)(x-3)}\}$$

is a linearly dependent subset of the space of continuous functions with domain $[0, 1]$.

4. (8 points) Let

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & -1 & 0 & 1 \\ -3 & 2 & 1 & -2 \\ 4 & 1 & 6 & 1 \end{bmatrix}$$

(a) (4 points) Find the reduced row-echelon form of A.

(b) (1 points) Find a basis of the row space of A and its dimension.

(c) (1 points) Find a basis of the column space of A and its dimension.

(d) (2 points) Find a basis of the null space of A and its dimension.

5. (6 points) Let β denote the second-last digit of your student number. Find a basis for the subspace of $\mathbb{P}_2$ given by

$$U = \{p(x) \in \mathbb{P}_2 : p(9 - \beta) = 0\}$$

and determine $\dim(U)$.