University of Ottawa

Department of Mathematics and Statistics

MAT 1341 B: Introduction to Linear Algebra

Instructor: Erhard Neher

Test 1; Sept. 27, 2007, 17:30-18:50

Family Name: ______________________________

First Name: ______________________________

Student number: ___________________________

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. You will get back the test without this first page.
- The table below is for the TA. Do not write in the table. For privacy reasons, this page of the test will be detached, and you will only get back the remaining pages of the test. Therefore, **fill in your name on both pages** and your student number on this page only.
- No books or notes are allowed. **Calculators are not permitted.**

Quest.	$1-6$	$7-8$	9	10	Total
maximal	12	4	5	7	28
score					

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. You may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
- For all questions you must show your work to obtain the points. Simply writing the correct answer will earn you 0.
- Please write legibly and argue logically: You must convince the TA that you know why your solution is correct.
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1. (2 points) Find the complex number z satisfying

$$(2+3i)z = (1-2i)(3+2i).$$

A. $z = \frac{2}{13} - \frac{29}{13}i$
B. $z = \frac{2}{13} + \frac{29}{13}i$
C. $z = 2 - 29i$
D. $z = 2 + i$
E. $z = 2 - i$
F. $z = \frac{2}{13} - \frac{1}{13}i$

My answer:______________________________

2. (2 points) Find all complex solutions z of the equation

$$z^2 - 6z + 34 = 0.$$

A. There are two solutions : $z = \frac{3}{2} + \frac{5}{2}i$ and $z = \frac{3}{2} - \frac{5}{2}i$
B. There are two solutions : $z = 3 + 5i$ and $z = 3 - 5i$
C. There are two solutions : $z = 1 + \frac{5}{2}i$ and $z = 1 - \frac{5}{2}i$
D. There are two solutions : $z = 2 + 5i$ and $z = 2 - 5i$
E. There is only solution : $z = 1 + \sqrt{2} + 5i$
F. There is only one solution : $z = 1 - \sqrt{2} + 5i$

My answer:______________________________

3. (2 points) Find the area of the triangle with vertices

$$A(2,1,-3), \quad B(1,2,-1), \quad \text{and} \quad C(1,1,0).$$

A. $\sqrt{91}$
B. $\sqrt{91}/2$
C. $\sqrt{91}/4$
D. $\sqrt{11}/8$
E. $\sqrt{11}/4$
F. $\sqrt{11}/2$

My answer:______________________________

4. (2 points) Which of the following is an equation for the plane containing the point $A(1,1,2)$ and the line with scalar equations $x = 3 + 2t$, $y = t$, $z = 1$?

A. $x + 3y - 2z = 1$
B. $x + 5y - 3z = 0$
C. $2x + y = 3$
D. $2x + 3y + z = 7$
E. $3x + z = 3$
F. $x - 2y + 4z = 7$

My answer:______________________________

5. (2 points) What is the shortest distance from the point $Q(1, -1, 1)$ to the plane with equation $3x + y - z = 3$?

A. $2\sqrt{11}$

B. $2/\sqrt{11}$

C. $2\sqrt{3}$

D. $2/\sqrt{3}$

E. $2/11$

F. 22

My answer:____________________________

6. (2 points) Which of the following lines goes through the two points $P(1, -2, 2)$ and $Q(2, 0, -1)$?

A. $\mathbf{p} = [1 \quad -2 \quad 2]^T + t\,[2 \quad 0 \quad -1]^T$

B. $\mathbf{p} = [3 \quad -2 \quad 1]^T + t\,[1 \quad 2 \quad -3]^T$

C. $\mathbf{p} = [3 \quad -2 \quad 1]^T + t\,[2 \quad 0 \quad -1]^T$

D. $\mathbf{p} = [2 \quad 0 \quad -1]^T + t\,[1 \quad 2 \quad 1]^T$

E. $\mathbf{p} = [3 \quad 2 \quad -4]^T + t\,[1 \quad 2 \quad -3]^T$

F. $\mathbf{p} = [3 \quad 2 \quad -4]^T + t\,[1 \quad 2 \quad 1]^T$

My answer:____________________________

7. (2 points) Find the matrix A satisfying

$$\left(2A^T - 3\begin{bmatrix} 4 & 0 \\ 4 & 3 \\ 0 & -1 \end{bmatrix}\right)^T = \begin{bmatrix} -4 & 0 & 4 \\ 5 & 7 & 0 \end{bmatrix}.$$

The first row of A is

A. $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$.

B. $[3 \quad -1 \quad 2]$.

C. $[4 \quad 6 \quad 2]$.

D. $\begin{bmatrix} 20 \\ 9 \end{bmatrix}$.

E. There does not exist such a matrix.

F. $[4 \quad 0 \quad -8]$.

My answer:______________________________

8. (2 points) In the matrix D below, replace α by the **last** digit of your student number. Consider the matrices

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & -5 & 0 & 3 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 1 & 2 & -3 & 5 \\ 0 & 0 & 3 & 0 & -2 \\ 1 & 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 & \alpha \end{bmatrix}, \quad E = \begin{bmatrix} 1 & -1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & -6 \end{bmatrix}.$$

Among the matrices A, B, C, D, E, the matrix/matrices in reduced row-echelon form is/are

My answer:______________________________

9. (5 points) Find the reduced row echelon form of the matrix

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 \\ 4 & -5 & 3 & 4 & -\beta \\ -3 & 2 & 0 & -8 & 2 \end{bmatrix}$$

where β is the second last digit of your student number, and determine its rank.

Answer: The reduced row echelon form is

The matrix has rank:

10. (7 points) Determine the values of a for which the linear system

$$\begin{array}{rcrcrcl} 3x & - & 6y & + & 5z & = & 9 \\ & & ay & - & 3z & = & 0 \\ & & & & a(a-1)z & = & a \end{array}$$

has (i) infinitely many solutions, (ii) a unique solution, (iii) no solution.
In case (i) determine all solutions.

Answer: Case (i): **Case (ii):** **Case (iii)**

All solutions in case (i):