University of Ottawa

Department of Mathematics and Statistics

MAT 1341 B: Introduction to Linear Algebra

Instructor: Erhard Neher

Assignment 5; due November 15, 2007, 17:30 in the class room

Family Name: ______________________

First Name: ______________________

Student number: ______________________

The last digit of your student number is $\alpha =$

The second last digit of your student number is $\beta =$

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the assignment.
- The table below is for the TA. Do not write in the table.

Quest.	1	2	3	4	5	Total
maximal	4	8	4	4	6	26
score						

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. You may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
- For all questions you must show your work to obtain the points. Simply writing the correct answer will earn you 0.
- Please write legibly and argue logically: You must convince the TA that you know why your solution is correct.
- You have to submit this assignment at the beginning of the DGD on Thursday, November 15, 2007, at 17:30 in the classroom, at the latest. If you wish to submit it earlier, please do so at the secretariat of the Department of Mathematics, room 103A, 8:45–12:00 and 13:00–17:00, or at the beginning of my Thursday lecture.

1. (4 points) (a) (2 points) Can a 3×4 matrix have linearly independent columns? Linearly independent rows?

(b) (2 points) Can the null space of a 3×6 matrix have dimension 2? Give a reasoned answer.

2. (8 points) Let α denote the **last digit** of your student number. Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 & 5 \\ 2 & 3 & 4 & 1 & 0 \\ -1 & -1 & -5 & 3 & -\alpha \\ 0 & 1 & -6 & 7 & 10 \end{bmatrix}.$$

(a) (2 points) Find the reduced row echelon form of A.
(b) (1 point) Find a basis for row(A).
(c) (1 point) Find a basis for col(A).
(d) (2 points) Find a basis for Null(A).
(e) (2 points) Give the dimensions of these three subspaces and verify the rank theorem (page 205) and the corollary about null space and image (page 209) for this matrix.

3. (4 points) Let $U = \{A \in \mathbb{M}_{4,4} : A^T = -A\}$.

(a) (1 point) Give an example of a non-zero matrix in U.

(b) (3 points) Show that U is a subspace of $\mathbb{M}_{4,4}$ by verifying the 3 conditions of the subspace test (Theorem 3 on page 297). Hint: You do not need to write out 4×4 matrices to answer this question. Instead, use matrix algebra (Chapter 1.1).

4. (4 points) Let U be the set of 2×2 non-invertible matrices:

$$U = \{A \in \mathbb{M}_{2,2} : \det(A) = 0\}$$

This is a subset of the vector space $\mathbb{M}_{2,2}$.
(a) (2 points) Give an example of two matrices which are in the set U but whose sum is not in U.
(b) (2 points) Is U a subspace of $\mathbb{M}_{2,2}$? Justify your answer.

5. (6 points) Let β be the **second last** digit of your student number. Let $\mathbb{P}_3$ denote the vector space consisting of all polynomials of degree at most 3. Let U be the set of polynomials in $\mathbb{P}_3$ which have β as a root, that is,

$$U = \{p(x) \in \mathbb{P}_3 : p(\beta) = 0\}.$$

Let V be the set of all polynomials in $\mathbb{P}_3$ such that when evaluated at 0 the answer is $\beta + 1$:

$$V = \{p(x) \in \mathbb{P}_3 : p(0) = \beta + 1\}$$

(a) Is U a subspace of $\mathbb{P}_3$? Justify your answer.
(b) Is V a subspace of $\mathbb{P}_3$? Justify your answer.