University of Ottawa

Department of Mathematics and Statistics

MAT 1341 B: Introduction to Linear Algebra

Instructor: Erhard Neher

Assignment 4; due November 8, 2007, 17:30 in the class room

Family Name: ______________________________

First Name: ______________________________

Student number: ___________________________

The last digit of your student number is $\alpha =$

The second last digit of your student number is $\beta =$

The third last digit of your student number is $\gamma =$

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the assignment.
- Question 3 is a more challenging question. This question does not count towards the total marks of the homework. You will get 5 extra points for a complete and correct solution.
- The table below is for the TA. Do not write in the table.

Quest.	1	2	3	Total
maximal	8	8	5 extra points	16
score				

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. You may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
- For all questions you must show your work to obtain the points. Simply writing the correct answer will earn you 0.
- Please write legibly and argue logically: You must convince the TA that you know why your solution is correct.
- You have to submit this assignment at the beginning of the DGD on Thursday, November 8, 2007, at 17:30 in the classroom, at the latest. If you wish to submit it earlier, please do so at the secretariat of the Department of Mathematics, room 103A, 8:45–12:00 and 13:00–17:00, or at the beginning of my Thursday lecture.

1. (8 points) For each of the following sets of vectors, answer two questions: (i) Do the vectors span $\mathbb{R}^3$? (ii) Is the set of vectors linearly independent? In each case, **justify your answer** and, if the vectors are linearly dependent, express one vector as a linear combination of the others. (Please note that a YES or NO answer to any question without a correct justification will not earn you any marks.)

(a) $\{[1,2,3]^T, [1,1,2]^T\}$
(b) $\{[1,2,3]^T, [1,1,2]^T, [2,1,0]^T\}$
(c) $\{[1,2,3]^T, [1,1,2]^T, [2,1,0]^T, [1,0,1]^T\}$

2. Let α, β, γ be the last, the second last and third last digit of your student number, and let

$$U = \{ [\,x_1 \quad x_2 \quad x_3 \quad x_4\,]^T : x_1 + \alpha x_2 + \beta x_3 + \gamma x_4 = 0 \text{ and } x_2 + \beta x_3 + \gamma x_4 = 0 \}.$$

(a) (3 points) Show that U is a subspace of $\mathbb{R}^4$.

(b) (4 points) Find a basis of U.

(c) (1 points) What is the dimension of U?

3. (**5 bonus points**) If $X, Y \in \mathbb{R}^4$ are two linearly independent vectors, show that there exist $s, t \in \{1, 2, 3, 4\}$ such that $\{X, Y, E_s, E_t\}$ is a basis of $\mathbb{R}^4$ (recall that $\{E_1, E_2, E_3, E_4\}$ is the standard basis of $\mathbb{R}^4$).