University of Ottawa

Department of Mathematics and Statistics

MAT 1341 B: Introduction to Linear Algebra

Instructor: Erhard Neher

Assignment 3; due October 18, 2007, 17:30 in the class room

Family Name: ____________________

First Name: ____________________

Student number: ____________________

The last digit of your student number is $\alpha =$

The second last digit of your student number is $\beta =$

Please read these instructions carefully:

- Enter your name on this page and the next, but your student number only on this page. For privacy reasons, this page of the assignment will be detached, and you will only get back the remaining pages of the assignment.
- Question 3 is a more challenging question. This question does not count towards the total marks of the homework. You will get 5 extra points for a complete and correct solution.
- The table below is for the TA. Do not write in the table.

Quest.	1	2	3	Total
maximal	5	10	5 extra points	15
score				

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Please read these instructions carefully:

- Read each question carefully, and answer all questions in the space provided after each question. You may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
- For all questions you must show your work to obtain the points. Simply writing the correct answer will earn you 0.
- Please write legibly and argue logically: You must convince the TA that you know why your solution is correct.
- You have to submit this assignment at the beginning of the DGD on Thursday, October 18, 2007, at 17:30 in the classroom, at the latest. If you wish to submit it earlier, please do so at the secretariat of the Department of Mathematics, room 103A, 8:45–12:00 and 13:00–17:00, or at the beginning of my Thursday lecture.

1. (5 points) In the matrix below replace β with the **second last** digit of your student number and calculate its determinant.

$$\begin{bmatrix} 3 & 5 & -2 & \beta \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}$$

My answer:____________________________

2. (10 points) In the matrix below replace α with the **last** digit of your student number.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 3\alpha & -5\alpha - 6 & -6\alpha - 6 \\ -3\alpha - 3 & 5\alpha + 5 & 6\alpha + 5 \end{bmatrix}.$$

(a) (3 points) Find the characteristic polynomial of A.

(b) (1 point) Find all eigenvalues of A.

(c) (4 points) For each eigenvalue find a set of basic eigenvectors.

(d) (2 points) Decide if A is diagonalizable or not. If yes, give an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$. Justify your answer.

3. (5 extra points) Let $x_1, x_2, \ldots, x_n$ be real numbers, $n \geq 2$. Show that

$$\begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{vmatrix} = \prod_{1 \leq j < i \leq n} (x_i - x_j)$$

where $\prod_{1 \leq j < i \leq n} (x_i - x_j)$ means the product of all factors $(x_i - x_j)$ for all pairs (i, j) satisfying $j < i$ and i and j between 1 and n. Hint: Replace the i^{th}-column C_i of the matrix in question by $C_i - x_1 C_{i-1}$, and expand along the first row. Then reduce to the analogous determinant for $n - 1$ numbers.