Unitary Space-Time Constellations Based on Finite Reflection Group Codes

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Abstract

This paper presents a design for full diversity unitary space-time constellations for two transmitter antennas. Our proposed constellations are based on finite reflection group codes. They have excellent diversity products compared to several unitary constellation designs in the literature.

I. INTRODUCTION

Consider a multiple-antenna system with M transmitter and N receiver antennas in a Rayleigh flat fading channel, using *unitary space-time modulation*, proposed in [3]. Let a signal constellation $\mathcal{V}$ be a set of L $M \times M$ unitary matrices. At high signal-to-noise ratio ρ, the pairwise error probability, P_e, that a receiver antenna decodes an error from $\boldsymbol{V}$ to $\boldsymbol{V}' \in \mathcal{V}$, can be approximated by [3], [4]

$$P_e \leq \frac{1}{2} \left(\frac{4\alpha}{\rho} \right)^{MN} \frac{1}{|\det(\boldsymbol{V} - \boldsymbol{V}')|^{2N}} \tag{1}$$

where $\alpha = 1$ and 2 for known and unknown channels (using *differential unitary space-time modulation* [4], [5]) respectively. The *diversity product* $\zeta_{\mathcal{V}}$ is computed from a signal constellation $\mathcal{V}$ via [7]

$$\zeta_{\mathcal{V}} = \frac{1}{2} \min_{\boldsymbol{V}, \boldsymbol{V}' \in \mathcal{V}} |\det(\boldsymbol{V} - \boldsymbol{V}')|^{\frac{1}{M}} \tag{2}$$

where the minimum is over all pairs $\boldsymbol{V}, \boldsymbol{V}' \in \mathcal{V}$ such that $\boldsymbol{V} \neq \boldsymbol{V}'$. Since each $\boldsymbol{V}$ is unitary, it follows that $0 \leq \zeta_{\mathcal{V}} \leq 1$. The design criterion of a unitary space-time constellation is to minimize

P_e in (1), and consequently, to find a unitary constellation $\mathcal{V}$ which has $\zeta_{\mathcal{V}}$ defined in (2) as large as possible. A unitary constellation $\mathcal{V}$ which has $\zeta_{\mathcal{V}} > 0$ is said to have full diversity.

We consider using a 2×2 Hamiltonian matrix to design a full diversity unitary constellation for two transmitter antennas as suggested in [7]. This matrix is defined by

$$\boldsymbol{V} = \begin{bmatrix} x & -y^* \\ y & x^* \end{bmatrix} \tag{3}$$

where $x, y \in \mathbb{C}$ and $|x|^2 + |y|^2 = 1$. We may thus identify a unitary constellation $\mathcal{V}$ consisting of Hamiltonian matrices with a set of points on the unit sphere in $\mathbb{R}^4$. Explicitly computing $\zeta_{\mathcal{V}}$ using (2) in this case yields $\zeta_{\mathcal{V}} = d_{\min}/2$, the minimum distance between two points in this set.

In this paper, we present the design of 2×2 Hamiltonian unitary space-time constellations based on 4-dimensional *finite reflection group codes* [6]. The constellations themselves are in general *not* groups, but lie in the larger class of *nongroup* unitary constellations. An advantage of using irreducible finite reflection group codes to construct unitary constellations in this way is that the problem of finding the best initial vector, and the value of $d_{\min}$, have been solved in [6] for all irreducible finite reflection groups. Consequently, the performance of the unitary constellations constructed from group codes may be predicted in advance. Having chosen a high-performance group code, the construction of the corresponding constellation is algorithmic and requires no optimization steps.

The paper is organized as follows: Section II gives a brief summary of finite reflection group codes. Our design is presented in Section III. Some examples of proposed constellations and their best diversity products based on irreducible finite reflection groups $\mathcal{B}_4$, $\mathcal{F}_4$ and $\mathcal{I}_4$ are also given in this section. One excellent proposed constellation obtained from the irreducible finite reflection group code $\mathcal{I}_4$ of $L = 120$ yields a diversity product equal to that of the excellent fixed-point free group $SL_2(\mathbb{F}_5)$, that is, $\zeta_{\mathcal{V}} = 0.3090$. This is the best known diversity product at the data rate $R = 3.45$ [7]. Section IV discusses conclusion and possible extension of this paper. Appendix shows how to use GAP [2] to generate all possible elements of a group.

II. FINITE REFLECTION GROUP CODES

Let $\mathcal{G}$ be a *finite reflection group* which can be generated by n-dimensional orthogonal reflection matrices, namely, the *reflections* $S_{\boldsymbol{r}}$, where $\boldsymbol{r} \in \mathbb{R}^n$ is a *simple root* of $\mathcal{G}$. The reflection

$S_{\boldsymbol{r}} \in \mathcal{G}$ is explicitly defined via [6]

$$S_{\boldsymbol{r}}\boldsymbol{x} = \boldsymbol{x} - 2\frac{\boldsymbol{x} \cdot \boldsymbol{r}}{\boldsymbol{r} \cdot \boldsymbol{r}}\boldsymbol{r} \tag{4}$$

for any $\boldsymbol{x} \in \mathbb{R}^4$. The n-dimensional finite reflection group codewords are generated from

$$\boldsymbol{c} = g \cdot \boldsymbol{x_0} \tag{5}$$

where $g \in \mathcal{G}$ of rank n and $\boldsymbol{x_0} \in \mathbb{R}^n$ is called an *initial vector*. If $\|\boldsymbol{x_0}\| = 1$, then each codeword will lie on the unit sphere of $\mathbb{R}^n$.

Now let $\Pi = \{\boldsymbol{r_1}, \boldsymbol{r_2}, \ldots, \boldsymbol{r_n}\}$ be a set of simple roots of $\mathcal{G}$. More generally, this is called a *root base* of $\mathcal{G}$, and has cardinality equal to the rank of $\mathcal{G}$. Given a carefully chosen initial vector $\boldsymbol{x_0}$, we define the corresponding *passive root base* [6] as $\Pi_p = \{\boldsymbol{r_i} \in \Pi : S_{\boldsymbol{r_i}}\boldsymbol{x_0} = \boldsymbol{x_0}\}$. Let $\mathcal{H}$ be the subgroup of $\mathcal{G}$ generated by the reflections in the roots of Π_p; then the group code derived from $\mathcal{G}$ and $\boldsymbol{x_0}$ has cardinality $|\boldsymbol{c}| = |\mathcal{G}|/|\mathcal{H}|$. This relationship of passive roots to the root base can be described most succinctly in terms of the *Coxeter graph* Γ of $\mathcal{G}$. Each subgraph $\{\Gamma_i\}_{i=1}^{l}$ (denoting a subgraph with l connected components) corresponds to a choice of passive root base $\Pi_p = \bigcup_{i=1}^{l} \Pi_{pi}$. In this case, $\mathcal{H} = \mathcal{G}_{\Gamma_1} \times \mathcal{G}_{\Gamma_2} \times \cdots \times \mathcal{G}_{\Gamma_l}$, where $\mathcal{G}_{\Gamma_i}$ is an irreducible finite reflection group corresponding to the subgraph Γ_i. Moreover, every finite reflection group code can be constructed in this way.

For a given minimum distance d (a nearest distance between two codewords), an optimal initial vector $\boldsymbol{x_0}$ is given by [6]

$$\boldsymbol{x_0} = \sum_{\boldsymbol{r_i} \in \Pi \setminus \Pi_p} \frac{d\|\boldsymbol{r_i}\|}{2}\boldsymbol{\lambda_i} \tag{6}$$

where $\boldsymbol{\lambda_i}$ is a dual basis vector of $\boldsymbol{r_i}$, that is, $\boldsymbol{\lambda_i} \cdot \boldsymbol{r_j} = \delta_{ij}$ for all $i, j \in \{1, 2, \ldots, n\}$. The minimum distance for a normalized initial vector $\|\boldsymbol{x_0}\| = 1$ is computed by

$$d_{\min}^2 = \frac{4}{\sum_{\boldsymbol{r_i} \in \Pi \setminus \Pi_p} \sum_{\boldsymbol{r_j} \in \Pi \setminus \Pi_p}(\|\boldsymbol{r_i}\| \cdot \|\boldsymbol{r_j}\|)\boldsymbol{\lambda_i} \cdot \boldsymbol{\lambda_j}}. \tag{7}$$

III. PROPOSED UNITARY CONSTELLATION DESIGN

The construction of our proposed unitary space-time constellations, based on finite reflection group codes, is presented in this section. For any codewords $\boldsymbol{c} = (c_1, c_2, c_3, c_4) \in \mathbb{R}^4$, we define a Hamiltonian matrix in (3) via

$$V_{\boldsymbol{c}} = \begin{bmatrix} c_1 + jc_2 & -(c_3 - jc_4) \\ c_3 + jc_4 & c_1 - jc_2 \end{bmatrix}. \tag{8}$$

Consider irreducible finite reflection groups $\mathcal{G}$ of rank $n = 4$, namely, $\mathcal{B}_4$, $\mathcal{I}_4$ or $\mathcal{F}_4$, and choose a normalized initial vector $||\boldsymbol{x_0} = 1|| \in \mathbb{R}^4$. Consequently the proposed 2×2 full diversity unitary constellation obtained from finite relfection group codes consists of all Hamiltonian matrices $V_{\boldsymbol{c}}$ in (8) with $\boldsymbol{c} = g \cdot \boldsymbol{x_0}$ for some $g \in \mathcal{G}$.

We present a method to compute the 4-dimensional orthogonal reflection matrix $S_{\boldsymbol{r}}$ as follows. Let an initial vector $\boldsymbol{x_0} = (x_1, x_2, x_3, x_4)$ and $\boldsymbol{r} = (r_1, r_2, r_3, r_4)$ be a root base of $\mathcal{G}$, an irreducible finite reflection group of rank 4. Using (4), the codeword generated from a reflection $S_{\boldsymbol{r}} \in \mathcal{G}$ from $\boldsymbol{x_0}$ is computed by

$$S_{\boldsymbol{r}}\boldsymbol{x_0} = \boldsymbol{x_0} - 2\frac{\boldsymbol{x_0} \cdot \boldsymbol{r}}{\boldsymbol{r} \cdot \boldsymbol{r}}\boldsymbol{r}$$

$$S_{\boldsymbol{r}}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 - 2\frac{\boldsymbol{x_0}\cdot\boldsymbol{r}}{\boldsymbol{r}\cdot\boldsymbol{r}}r_1 \\ x_2 - 2\frac{\boldsymbol{x_0}\cdot\boldsymbol{r}}{\boldsymbol{r}\cdot\boldsymbol{r}}r_2 \\ x_3 - 2\frac{\boldsymbol{x_0}\cdot\boldsymbol{r}}{\boldsymbol{r}\cdot\boldsymbol{r}}r_3 \\ x_4 - 2\frac{\boldsymbol{x_0}\cdot\boldsymbol{r}}{\boldsymbol{r}\cdot\boldsymbol{r}}r_4 \end{bmatrix}$$

$$= \begin{bmatrix} x_1\left(1 - \frac{2r_1}{\boldsymbol{r}\cdot\boldsymbol{r}}\right) - \frac{2x_2r_1}{\boldsymbol{r}\cdot\boldsymbol{r}} - \frac{2x_3r_1}{\boldsymbol{r}\cdot\boldsymbol{r}} - \frac{2x_4r_1}{\boldsymbol{r}\cdot\boldsymbol{r}} \\ \frac{2x_1r_2}{\boldsymbol{r}\cdot\boldsymbol{r}} + x_2\left(1 - \frac{2r_2}{\boldsymbol{r}\cdot\boldsymbol{r}}\right) - \frac{2x_3r_2}{\boldsymbol{r}\cdot\boldsymbol{r}} - \frac{2x_4r_2}{\boldsymbol{r}\cdot\boldsymbol{r}} \\ -\frac{2x_1r_3}{\boldsymbol{r}\cdot\boldsymbol{r}} - \frac{2x_2r_3}{\boldsymbol{r}\cdot\boldsymbol{r}} + x_3\left(1 - \frac{2r_3}{\boldsymbol{r}\cdot\boldsymbol{r}}\right) - \frac{2x_4r_3}{\boldsymbol{r}\cdot\boldsymbol{r}} \\ -\frac{2x_2r_4}{\boldsymbol{r}\cdot\boldsymbol{r}} - \frac{2x_2r_4}{\boldsymbol{r}\cdot\boldsymbol{r}} - \frac{2x_3r_4}{\boldsymbol{r}\cdot\boldsymbol{r}} + x_4\left(1 - \frac{2r_4}{\boldsymbol{r}\cdot\boldsymbol{r}}\right) \end{bmatrix}. \tag{9}$$

Solve (9), hence , for a given root base $\boldsymbol{r} = (r_1, r_2, r_3, r_4)$, the 4-dimensional orthogonal reflection matrix is

$$S_{\boldsymbol{r}} = \begin{bmatrix} 1 - p_1r_1 & -p_1r_2 & -p_1r_3 & -p_1r_4 \\ -p_2r_1 & 1 - p_2r_2 & -p_2r_3 & -p_2r_4 \\ -p_3r_1 & -p_3r_2 & 1 - p_3r_3 & -p_3r_4 \\ -p_4r_1 & -p_4r_2 & -p_4r_3 & 1 - p_4r_4 \end{bmatrix} \tag{10}$$

where $p_i = \frac{2r_i}{\boldsymbol{r}\cdot\boldsymbol{r}}$, $i = 1, 2, 3, 4$.

The codewords $\boldsymbol{c}$ of the group code (and hence, via the formula (8), the elements of the constellation $\mathcal{V}$) are thus obtained from $\boldsymbol{x_0}$ by

$$\boldsymbol{c} = \prod_{i_j} S_{\boldsymbol{r}_{i_j}}\boldsymbol{x_0} \tag{11}$$

where the indices $i_j \in \{1, 2, 3, 4\}$ are determined by the presentation of the finite reflection group (using, for example, [2]), and $S_{\boldsymbol{r}_i}$ is defined in (10). The data rate is $R = \log_2 |\boldsymbol{c}|/2$

bits/s/Hz. From (7), finally, the diversity product of proposed unitary constellation obtained from an irreducible finite reflection group code defined in (8) is

$$\zeta_{\mathcal{V}} = \left[\sum_{\boldsymbol{r_i} \in \Pi \backslash \Pi_p} \sum_{\boldsymbol{r_j} \in \Pi \backslash \Pi_p} (||\boldsymbol{r_i}|| \cdot ||\boldsymbol{r_j}||) \boldsymbol{\lambda_i} \cdot \boldsymbol{\lambda_j} \right]^{-\frac{1}{2}}. \tag{12}$$

A. Proposed Constellaiton From $\mathcal{I}_4$

Fig. 1.   The coxeter graph of the irreducible finite reflection group $\mathcal{I}_4$

We give one example of constructing a proposed unitary constellation based on an irreducible finite reflection group $\mathcal{I}_4$ with its coxeter graph shown in Figure 1. The root base $\Pi = \{\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3}, \boldsymbol{r_4}\}$ of $\mathcal{I}_4$ consists of the 4 simple roots [6]

$$\begin{aligned} \boldsymbol{r_1} &= \cos\frac{2\pi}{5}\left(2\cos\frac{\pi}{5} + 1, 1, -2\cos\frac{\pi}{5}, 0\right) \\ \boldsymbol{r_2} &= \cos\frac{2\pi}{5}\left(-2\cos\frac{\pi}{5} - 1, 1, 2\cos\frac{\pi}{5}, 0\right) \\ \boldsymbol{r_3} &= \cos\frac{2\pi}{5}\left(2\cos\frac{\pi}{5}, -2\cos\frac{\pi}{5} - 1, 1, 0\right) \\ \boldsymbol{r_4} &= \cos\frac{2\pi}{5}\left(-2\cos\frac{\pi}{5}, 0, -2\cos\frac{\pi}{5} - 1, 1\right). \end{aligned}$$

From its coxeter graph, the total elements of $\mathcal{I}_4$ can be expressed as $\mathcal{I}_4 = \{S_{\boldsymbol{r_1}}^2 = S_{\boldsymbol{r_2}}^2 = S_{\boldsymbol{r_3}}^2 = S_{\boldsymbol{r_4}}^2 (S_{\boldsymbol{r_1}}S_{\boldsymbol{r_2}})^5 = (S_{\boldsymbol{r_1}}S_{\boldsymbol{r_3}})^2 = (S_{\boldsymbol{r_1}}S_{\boldsymbol{r_4}})^2 = (S_{\boldsymbol{r_2}}S_{\boldsymbol{r_3}})^3 = (S_{\boldsymbol{r_2}}S_{\boldsymbol{r_4}})^2 = (S_{\boldsymbol{r_3}}S_{\boldsymbol{r_4}})^3 = I\}$ with its cardinality $|\mathcal{I}_4| = 14400$. Using (10), the 4-dimensional orthogonal reflection matrices of

these simple roots r_1, r_2, r_3 and r_4 are

$$S_{r_1} = \begin{bmatrix} -\cos\frac{2\pi}{5} & -\frac{1}{2} & \cos\frac{\pi}{5} & 0 \\ -\frac{1}{2} & \cos\frac{\pi}{5} & \cos\frac{2\pi}{5} & 0 \\ \cos\frac{\pi}{5} & \cos\frac{2\pi}{5} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{r_2} = \begin{bmatrix} -\cos\frac{2\pi}{5} & \frac{1}{2} & \cos\frac{\pi}{5} & 0 \\ \frac{1}{2} & \cos\frac{\pi}{5} & -\cos\frac{2\pi}{5} & 0 \\ \cos\frac{\pi}{5} & -\cos\frac{2\pi}{5} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{r_3} = \begin{bmatrix} \frac{1}{2} & \cos\frac{\pi}{5} & -\cos\frac{2\pi}{5} & 0 \\ \cos\frac{\pi}{5} & -\cos\frac{2\pi}{5} & \frac{1}{2} & 0 \\ -\cos\frac{2\pi}{5} & \frac{1}{2} & \cos\frac{\pi}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{r_4} = \begin{bmatrix} \frac{1}{2} & 0 & -\cos\frac{\pi}{5} & \cos\frac{2\pi}{5} \\ 0 & 1 & 0 & 0 \\ -\cos\frac{\pi}{5} & 0 & -\cos\frac{2\pi}{5} & \frac{1}{2} \\ \cos\frac{2\pi}{5} & 0 & \frac{1}{2} & \cos\frac{\pi}{5} \end{bmatrix}$$

respectively. Let us choose an initial vector $x_0 = (0,0,0,1)$, which yields a passive root $\Pi_p = \{r_1, r_2, r_3\}$ since $S_{r_i} x_0 = x_0$ for $i = 1, 2$ and 3. This gives a constellation of cardinality 14400/120=120 which has data rate $R = \log_2 120/2 \approx 3.45$ bits/s/Hz (for a given passive root base Pi_p, we use GAP [2] to compute $\prod_{i_j} S_{r_{i_j}}$ in (11) to generate all possible codewords, see Appendix). Table I gives the comparison of this proposed constellation with different 2×2 unitary constellation designs at the same data rate $R \approx 3.45$ bits/s/Hz. We can see that our proposed constellation has the highest diversity product $\zeta_\mathcal{V} = 0.3090$, which equals the excellent unitary group constellation obtained from a fixed-point free group $SL_2(\mathbb{F}_5)$ [7].

Table II shows some examples of proposed unitary constellations obtained from 4-dimensional irreducible finite reflection groups $\mathcal{B}_4, \mathcal{F}_4$ and $\mathcal{I}_4$ with their best diversity products.

Unitary space-time constellation designs	L	R	$\zeta_\mathcal{V}$
Dicyclic group [5]	128	3.50	0.0491
Cyclic group [4]	120	3.45	0.1353
Orthogonal [1], [8]	121	3.46	0.1992
Our proposed code from $\mathcal{I}_4$	120	3.45	0.3090

TABLE I

OUR PROPOSED CONSTELLATION IS HIGHLIGHTED IN GREY.

Irreducible finite reflection groups	L	R	$\zeta_\mathcal{V}$
$\mathcal{F}_4$ $\boldsymbol{r_1, r_2, r_3} \in \Pi_p$	24	2.29	0.5000
$\mathcal{B}_4$ $\boldsymbol{r_2, r_3} \in \Pi_p$	64	3.00	0.3363
$\mathcal{B}_4$ $\boldsymbol{r_1, r_3} \in \Pi_p$	96	3.29	0.2861
$\mathcal{F}_4$ $\boldsymbol{r_1, r_2, r_4} \in \Pi_p$	96	3.29	0.2887
$\mathcal{B}_4$ $\boldsymbol{r_2} \in \Pi_p$	192	3.79	0.2107
$\mathcal{I}_4$ $\boldsymbol{r_1, r_2, r_3} \in \Pi_p$	120	3.45	0.3090
$\mathcal{F}_4$ $\boldsymbol{r_2, r_3} \in \Pi_p$	144	3.59	0.2706
$\mathcal{F}_4$ $\boldsymbol{r_1, r_2} \in \Pi_p$	192	3.79	0.1890
$\mathcal{F}_4$ $\boldsymbol{r_1, r_3} \in \Pi_p$	288	4.09	0.1913
$\mathcal{F}_4$ $\boldsymbol{r_2} \in \Pi_p$	576	4.59	0.1429
$\mathcal{I}_4$ $\boldsymbol{r_2, r_3, r_4} \in \Pi_p$	600	4.61	0.1350
$\mathcal{I}_4$ $\boldsymbol{r_1, r_2, r_4} \in \Pi_p$	720	4.75	0.1625
$\mathcal{F}_4$ $\emptyset \in \Pi_p$	1152	5.09	0.0967
$\mathcal{I}_4$ $\boldsymbol{r_1, r_3, r_4} \in \Pi_p$	1200	5.11	0.1102
$\mathcal{I}_4$ $\boldsymbol{r_1, r_2} \in \Pi_p$	1440	5.25	0.1077
$\mathcal{I}_4$ $\boldsymbol{r_2, r_3} \in \Pi_p$	2400	5.61	0.0955
$\mathcal{I}_4$ $\boldsymbol{r_1, r_3} \in \Pi_p$	3600	5.91	0.0823
$\mathcal{I}_4$ $\boldsymbol{r_2} \in \Pi_p$	7200	6.41	0.0604
$\mathcal{I}_4$ $\emptyset \in \Pi_p$	14400	6.91	0.0391

TABLE II

SOME EXAMPLE OF PROPOSED CONSTELLATIONS

IV. CONCLUSION

We have presented a design of full diversity unitary space-time constellations based on 4-dimensional irreducible finite reflection groups, namely, $\mathcal{B}_4, \mathcal{F}_4$ and $\mathcal{I}_4$. They have excellent

diversity products and outperform compared to different unitary constellation designs in the literature. The permutation-decoding algorithm proposed in [6] for finite reflection group codes can be considered as extension work, a low complexity decoding scheme of proposed constellations.

APPENDIX
USING GAP TO GENERATE ELEMENTS OF GROUP

Consider an irreducible finite reflection group $\mathcal{I}_4$. It has 4 simple roots r_1, r_2, r_3 and r_4, and its coxeter graph is shown in Figure 1. We first define a free group `G` acting on these 4 simple roots by `G:=FreeGroup("r1","r2","r3","r4");`. Then, from knowing the relations of 4 simple roots in Coxeter graph, the irreducible finite reflection group $\mathcal{I}_4$ can be defined using `I4:=G/[G.1^2,G.2^2,G.3^2,G.4^2, (G.1*G.2)^5,(G.1*G.3)^2,(G.1*G.4)^2, (G.2*G.3)^3,(G.`. The subgroup `H` generated from a passive root $\Pi_p = \{r_1, r_2, r_3\}$ is `H:=Subgroup(I4,[I4.1,I4.2,I4.3`. Consequently the indices i_j of $\prod_{i_j} S_{r_{i_j}}$ in (11) obtained from these 3 passive roots are listed by `LeftCosets(I4,H);`.

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