Unitary Space-Time Codes from Group Codes: Permutation Codes Variant II

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Abstract

Space-time coding was developed for use in multiple-antenna wireless communications to achieve high data rate and low error probability. The design of a full diversity space-time code with high coding gain and simple encoding-decoding algorithm poses a particularly challenging problem.

In this paper, we present new unitary space-time codes based on permutation codes variant II for differential unitary space-time modulations. Our codes have full diversity, and are used with two transmitter antennas for both known and unknown channels. Simulation results show that proposed codes have excellent performance at high signal-to-noise ratio.

Keywords: multiple-antenna systems, differential unitary space-time modulations, permutation codes variant II

1 Introduction

In practical applications and without imposing any training schemes, the fading coefficient of the channel is generally not known to the receiver antenna. Differential unitary space-time modulations [2, 3] were proposed for use with unknown fading channels, in which neither the transmitter nor the receiver antennas know the information of the channels. Let M be the number of transmitter antennas, and $\mathcal{V} = \{V_l\}_{l=0}^{L-1}$ be a signal unitary constellation. The design problem of differential unitary space-time modulation is to maximize the diversity product, $\zeta_{\mathcal{V}}$, which is defined by

$$\zeta_{\mathcal{V}} = \frac{1}{2} \min_{0 \leq l < l' \leq L-1} |\det(V_l - V_{l'})|^{\frac{1}{M}} \geq 0. \tag{1}$$

A constellation $\mathcal{V}$ which has $\zeta_{\mathcal{V}} > 0$ is said to have full diversity.

In this paper, we present new full diversity unitary constellations with high diversity product from permutation codes variant II [5] for differential space-time modulations with two transmitter antennas. This paper is organized as follows: Section 2 briefly reviews differential unitary space-time modulations. Section 3 discusses the idea of permutation codes variant II. Section 4 presents and lists full diversity unitary constellation designs from permutation codes variant II. Performance of some proposed constellations comparing with different designs is shown in Section 5. Section 6 concludes and gives possible extensions of this paper.

2 Differential Unitary Space-Time Modulation

Consider a multiple-antenna system in a Rayleigh flat fading channel with M transmitter and N receiver antennas. The $M \times N$ received signal matrix X_τ is

$$X_\tau = \sqrt{\rho} S_\tau H_\tau + W_\tau, \qquad \tau = 0, 1, \ldots \tag{2}$$

where S_τ is the $M \times M$ transmitted signal matrix, W_τ is the $M \times N$ additive noise matrix, H_τ is the $M \times N$ channel matrix, and ρ is the signal-to-noise ratio (SNR) at each receiver antenna. The fading coefficient of a channel matrix and the additive noise on receiver antenna are independent complex Gaussian variables with mean zero and variance one, $\mathcal{CN}(0, 1)$. Let V_{z_τ} be an $M \times M$ unitary message matrix at block τ which is chosen from a message $z_\tau \in \{0, 1, \ldots, L-1\}$. The date rate is $R = \log_2 L/M$. In differential unitary space-time modulation, the transmitted signal matrix is

$$S_\tau = V_{z_\tau} S_{\tau-1}, \tag{3}$$

with $S_0 = I_M$. Also we assume that the channel matrix is constant over two consecutive time periods, that is $H_\tau \approx H_{\tau-1}$. Then the received signal matrix of (2) can be written as

$$X_\tau = V_{z_\tau} X_{\tau-1} + \sqrt{2} W'_\tau \tag{4}$$

where W'_τ is also independent $\mathcal{CN}(0, 1)$. The maximum-likelihood (ML) decoder is used for the receiver antenna to decide a message $\hat{z}_\tau$ to be

$$\hat{z}_\tau = \arg \min_{l=0,1,\ldots,L-1} \|X_\tau - V_l X_{\tau-1}\|. \tag{5}$$

3 Permutation Codes Variant II

We introduce permutation codes variant II as proposed in [5] in this section. Consider the Euclidean space of dimension n. The codewords of a permutation code variant II, $\{X_i\}_{i=0}^{L-1}$, are all of the form

$$\{X_i\}_{i=0}^{L-1} = (\pm\mu_1, \pm\mu_1, \ldots, \pm\mu_1, \pm\mu_2, \pm\mu_2 \ldots, \pm\mu_2, \ldots, \pm\mu_k, \ldots, \pm\mu_k)^P \tag{6}$$

where $\{\mu_i\}_{i=1}^{k} \in \mathbb{R}$ and $0 < \mu_1 < \mu_2 < \ldots < \mu_k$. $(\ )^P$ and $\pm$ denote all possible permutation and combination of sign changes of the components in X_i respectively. Thus the order of permutation code variant II is

$$L = \frac{(m_1 + m_2 + \cdots + m_k)!}{m_1!\ m_2!\ \cdots m_k!} \times 2^{m_1+m_2+\cdots+m_k} \tag{7}$$

where μ_i appears m_i times in X_i. The first and the second terms in (7) are from possible permutation and combination of sign changes in X_i respectively. We call $m = (m_1, m_2, \ldots, m_k)$ *a distribution* of codewords. The minimum square distance of codewords can be computed as

$$d_{\min}^2 = \min\{\min_{i \neq j} 2(\mu_i - \mu_j)^2, 4\mu_1^2\}, \qquad i, j = 1, 2, \ldots, k. \tag{8}$$

The term $\min_{i \neq j} 2(\mu_i - \mu_j)^2$ is from the nearest distance of two different codewords, and the term $4\mu_1^2$ is from the same component codeword with one different sign at μ_1. The energy of a codeword, E, is

$$E = \sum_{i=1}^{k} m_i \mu_i^2. \tag{9}$$

4 Full Diversity Unitary constellation designs

We define a 2×2 full diversity unitary space-time constellation using for two transmitter antennas as

$$V(a_1, a_2, a_3, a_4) = \begin{bmatrix} a_1 + ja_2 & -(a_3 - ja_4) \\ a_3 + ja_4 & a_1 - ja_2 \end{bmatrix}. \tag{10}$$

$(a_1, a_2, a_3, a_4) \in \mathcal{A}$, where $\mathcal{A}$ is a permutation code variant II normalized by its energy $\sqrt{E}$, that is $\mathcal{A} = \frac{1}{\sqrt{E}}\{X_i\}_{i=0}^{L-1}$. The order of a constellation is equal to L in (7). For a given $\sum_{i=1}^{k} m_i = 4$, the diversity product of a constellation is maximized by choosing

$$m_1 > m_2 > \ldots > m_k \tag{11}$$

and

$$\mu_i = \frac{1}{\sqrt{2}} + i - 1. \tag{12}$$

The condition of μ_i in (12) makes the minimum square distance of codewords $d_{\min}^2$ of (8) =2. Then the diversity product of proposed constellation is (see a proof in Appendix A)

$$\zeta_{\mathcal{V}} = \frac{1}{\sqrt{2E}}. \tag{13}$$

The energy $E = \sum_{i=1}^{k} m_i \mu_i^2$ where μ_i is defined in (12). Table 1 lists our proposed constellations with their best diversity products and the distribution m.

5 Performance

We compare the performance of proposed constellations as listed Table 1 with different 2×2 unitary space-time constellation designs using for two transmitter antennas, $M = 2$. The performance is considered by

L	$\zeta_{\mathcal{V}}$	m	
16	0.5000	4	
64	0.3367	(3,1)	Figure 1
96	0.2706	(2,2)	
192	0.2109	(2,1,1)	Figure **??**
384	0.1429	(1,1,1,1)	

Table 1: Our proposed constellations with their best diversity products and distributions

plotting the block error rate, *bler*, against SNR. All plots are considered in unknown Rayleigh flat fading channels which receiver antenna does not know the information of channels by using differential unitary space-time modulations as explained in Section 2. The ML decoder of (5) is used for decoding.

- $\mathbf{R = 3.00, N = 2}$: From Table 1, our proposed constellation at $L = 64,\ R = 3.00$ has $m = (3, 1)$. Hence the energy is $E = 3 \times (\frac{1}{\sqrt{2}})^2 + 1 \times (\frac{1}{\sqrt{2}} + 1)^2 = 4.4142$. This gives the alphabet $\mathcal{A}$ of a constellation is

$$\mathcal{A} = \left\{ \frac{1}{\sqrt{4.4142}} \left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, \pm(\frac{1}{\sqrt{2}} + 1) \right)^P \right\}. \tag{14}$$

The diversity product is $\zeta_{\mathcal{V}} = 1/\sqrt{2 \times 4.4142} = 0.3366$ which is the largest diversity product among different 2×2 unitary space-time constellation designs at the same data rate $R = 3.00$ as listed in Table 2. Figure 1 shows the block error rate performance of our constellation with two receiver antennas, $N = 2$. We can see that our proposed code performs very well for high SNR, and outperforms other four designs.

- $\mathbf{R \approx 4.00, N = 1}$: From Table 1, the distribution of our constellation at $L = 192,\ R = 3.79$ is $m = (2, 1, 1)$. Hence $E = 2 \times (\frac{1}{\sqrt{2}})^2 + 1 \times (\frac{1}{\sqrt{2}} + 1)^2 + 1 \times (\frac{1}{\sqrt{2}} + 2)^2 = 11.2426$. A constellation has the alphabet

$$\mathcal{A} = \left\{ \frac{1}{\sqrt{11.2426}} \left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, \pm(\frac{1}{\sqrt{2}} + 1), \pm(\frac{1}{\sqrt{2}} + 2) \right)^P \right\}. \tag{15}$$

with the diversity product $\zeta_{\mathcal{V}} = 1/\sqrt{2 \times 11.2426} = 0.2109$. Table 3 gives a comparison of the diversity products of proposed constellation, cyclic, dicyclic groups and orthogonal designs at the data rate $R \approx 4.00$. Our constellation has the largest diversity product, and also outperforms other three designs as displayed in Figure **??**.

Unitary space-time code designs	$\zeta_{\mathcal{V}}$
Dicyclic group Q_5 [3]	0.0980
Cyclic group $u = (1, 19)$ [2]	0.1985
Orthogonal with 8^{th}-roots of unity [6]	0.2706
Parametric code, $k = (7, 10, 0)$ [4]	0.3070
Numerical method $A^k B^k$ [1]	0.3090
Our proposed code	0.3366

Table 2: Comparison of different 2×2 unitary constellation designs for $R = 3.00$. Our proposed code is highlighted in grey

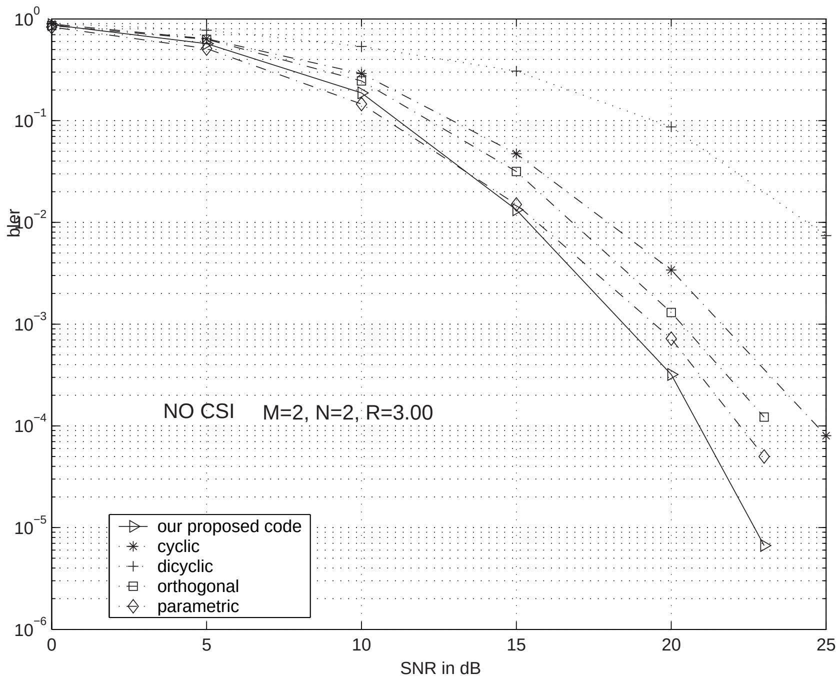

Figure 1: Block error rate performance for $M = 2,\ N = 2$ at $R = 3.00$ of our proposed code, dicyclic group, cyclic group, orthogonal and parametric code designs

6 Conclusion

We have constructed

Unitary space-time code designs	R	$\zeta_{\mathcal{V}}$
Dicyclic group Q_7 [3]	4.00	0.0980
Cyclic group $u = (1, 151)$ [2]	3.95	0.1985
Orthogonal with 16^{th}-roots of unity [6]	4.00	0.1379
Our proposed code	3.79	0.2109

Table 3: Comparison of different 2×2 unitary constellation designs for $R \approx 4.00$. Our proposed code is highlighted in grey

Appendix A: Proof of Diversity Product

We prove that the diversity product of proposed constellation is $\zeta_{\mathcal{V}} = \frac{1}{\sqrt{2E}}$. From the definition of $\zeta_{\mathcal{V}}$ in (1), we have

$$
\begin{aligned}
\zeta_{\mathcal{V}} &= \frac{1}{2}|\det(V - V')|^{\frac{1}{2}} \\
&= \frac{1}{2}\left|\det\begin{bmatrix} (a_1 - a_1') + j(a_2 - a_2') & -((a_3 - a_3') + j(a_4 - a_4'))^* \\ (a_3 - a_3') + j(a_4 - a_4') & ((a_1 - a_1') + j(a_2 - a_2'))^* \end{bmatrix}\right|^{\frac{1}{2}} \\
&= \frac{1}{2}\sqrt{(a_1 - a_1')^2 + (a_2 - a_2')^2 + (a_3 - a_3')^2 + (a_4 - a_4')^2} \\
&= \frac{1}{2}\sqrt{\frac{d_{\min}^2}{E}} \qquad \text{from } a_1, a_2, a_3, a_4 \in \mathcal{A} = \frac{1}{E}\{X_i\}_0^{L-1} \\
&= \frac{1}{2}\sqrt{\frac{2}{E}} \qquad \text{from the condition of } \mu_i \text{ in 12} \\
&= \frac{1}{\sqrt{2E}}
\end{aligned}
$$

□

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