

So since

$$3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and not all coefficients are zero, the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \right\}$ is **linearly dependent**.

What about the opposite? Linearly independent
what we like to have

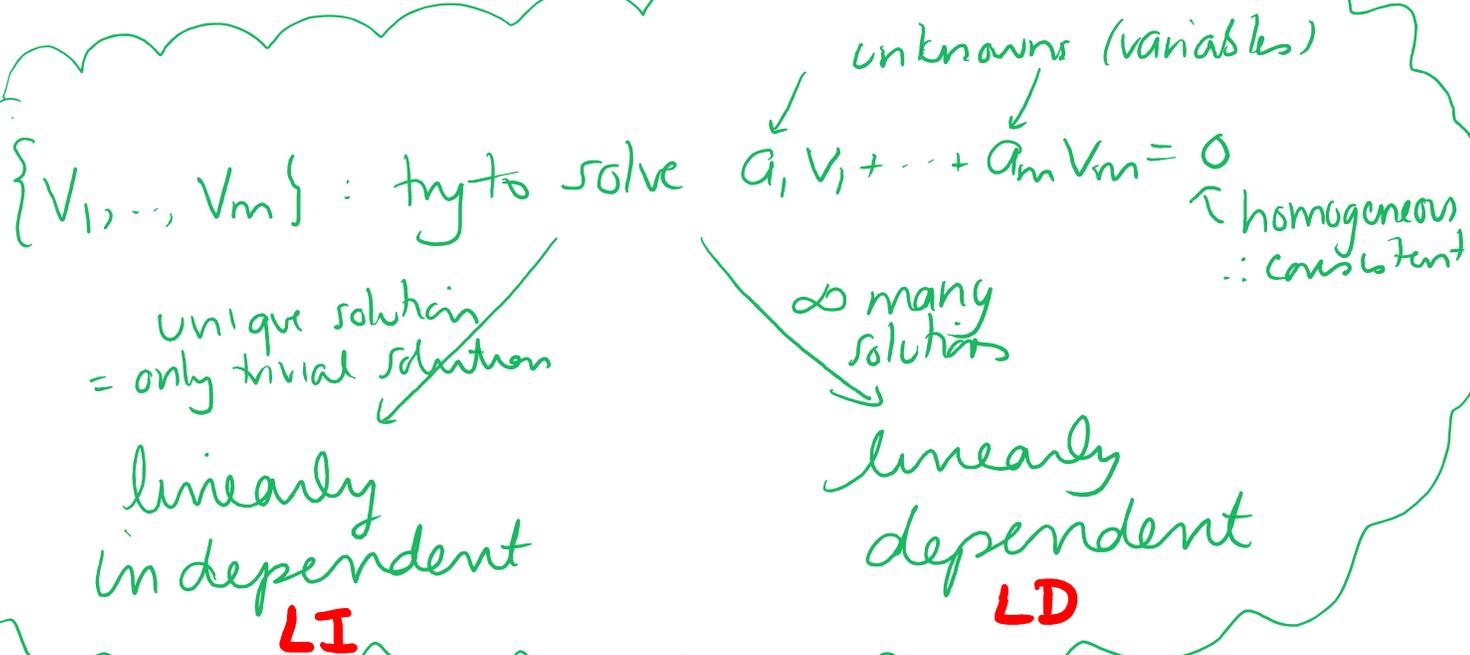
Defn: Let V be a vector space and let $v_1, \dots, v_m \in V$.

Then the set $\{v_1, \dots, v_m\}$ is **linearly independent**

if and only if the **ONLY** solution to the **dependence equation** is the **trivial solution**. That is, when you

solve $a_1 v_1 + \dots + a_m v_m = 0$

the only solution is $a_1 = 0, a_2 = 0, \dots, a_m = 0$.



Examples: Decide if the following sets are **LI** or **LD**.

1) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$: solve $a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Leftrightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

so the only solution is $a=0, b=0$
 \therefore this set is **LI**.

2) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$: solve $a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Leftrightarrow \begin{bmatrix} a+c \\ b-c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This has ∞ many solutions; for example: $\underbrace{a=1 \ c=-1 \ b=-1}_{\text{not all 0}}$
 gives $\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

\therefore this set is **LD**.

3) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$: solve $a \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$! The only solution is $a=0$.

\therefore this set is **LI**.

4) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$: solve $a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Leftrightarrow \begin{bmatrix} a & b \\ -b & a+c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow \begin{matrix} a=0 & b=0 \\ a+c=0 \Leftrightarrow c=0 \end{matrix}$$

\therefore the only solution is the trivial solution

\therefore this set is **LI**.

5) $\{1, x, x^2\} \subseteq P_2$: solve $a1 + bx + cx^2 = 0$

$\Leftrightarrow a=0 \ b=0 \ c=0$. This set is \therefore **LI**

6) $\{1+x^2, 1-x^2, x^2\}$ Notice that

$$(1+x^2) - (1-x^2) - 2(x^2) = 0$$

& not all coefficients are zero \therefore this set is **LD**.

0 polynomial