Université d'Ottawa · University of Ottawa
Faculté des sciences Mathématiques et de statistique · Faculty of Science Mathematics and Statistics

MAT 2377
Solutions to the Mi-term

Tuesday June 16 2015 **Professor M. Alvo**

Time: 70 minutes
Student **Number:**________________________
Name: ______________________________

This is an open book exam. Standard calculators are permitted**.** Answer all questions. **Place your answers in the table below and remit the entire exam.**

The answers in order for the questions below are

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	B	A	C	B	D	B	A	D	D	A	C	B	C	E	B

1. A , B and C are three independent events such that $P(A) = .5$, $P(B) = .2$ and $P(C) = .3$. What is the probability that at least one of the events occurs?

   (A) $.03$ (B) $.72$ (C) 0 (D) 1 (E) $.5$

$P(A \cup B \cup C) = 1 - P(A' \cap B' \cap \cap C') = 1 - P(A')P(B')P(C') = 1 - .05(.08)(.07) = 0.72$

1. A President and a treasurer are to be chosen from a group of 20 students. The President and treasurer must be two different individuals. (i) In how many ways can the positions be filled if there are no other restrictions ? (ii) In how many ways can the positions be filled if the students labeled A and B will not serve together?

   On choisit un president et un tresaurier a partir d'un groupe de 20 etudiants. Ils doivent etre deux etudiants differents. (i) Dans combien de facons peut-on combler les deux postes s'il n'y a pas de restrictions? (ii) Dans combien de facons peut-on combler les deux postes si les deux etudiants nommer A et B ne peuvent pas servir ensemble?

   (A) $\begin{matrix} (i) & 380 \\ (ii) & 378 \end{matrix}$ (B) $\begin{matrix} (i) & 400 \\ (ii) & 398 \end{matrix}$ (C) $\begin{matrix} (i) & 400 \\ (ii) & 380 \end{matrix}$ (D) $\begin{matrix} (i) & 380 \\ (ii) & 190 \end{matrix}$

   (E) $\begin{matrix} (i) & 400 \\ (ii) & 2 \end{matrix}$

   See example 1.16 p.21

i) 20(19)=380 ii)380-2=378 where 2 is the number of ways they serve together

1. A scientist believes that one male mouse in 20 and one female mouse in 500 is colorblind. A mouse is chosen at random and found to be colorblind. Given that male and female mice are equally distributed in the population of mice, what is the probability that it is female?

   (A) $\frac{1}{500}$ (B) $\frac{500}{520}$ (C) $\frac{1}{26}$ (D) $\frac{1}{20}$ (E) $\frac{25}{26}$

   This is a direct application of Bayes theorem

Let M be the event of a male mouse and B be the event of colorblind. Then we are given

$$P(M) = 0.5, P(M') = 0.5,$$
$$P(B|M) = 1/20, P(B|M') = 1/500$$

Hence, $P\left(M'|B\right)=\frac{P(B|M")P(M')}{P(B|M)P(M)+P(B|M')P(M')}=\frac{0.5(1/500))}{0.5(1/20)+0.5(1/500)}=1/26$

1. The probability that a randomly chosen number in the interval (0,1) is less than 0.2 is 0.2. We choose 5 numbers at random in the interval (0,1). What is the probability that exactly 3 of them are less than 0.2?
   (A) 0.9421 (B) 0.0512 (C) 0.2048 (D) 0.0579 (E) $(0.8)^3$
   This is a direct application of the Binomial with parameters n=5 and p=0.2.

P(exactly 3 successes)=F(3)-F(2)=0.9933-0.9421=0.0512

1. I have 5 keys only one of which opens the door to my office. Every morning, I try one key after the other until I find the correct one. What is the probability that the correct key is the third one I try?

   J'ai 5 clés qui se ressemblent dont une seule ouvre la porte de mon bureau. Tous les matins j'essaie une clé après l'autre jusqu'à ce que je trouve la bonne clé pour entrer dans mon bureau. Quelle est la probabilité que la bonne clé soit la troisième clé choisie?

   (A) $\frac{16}{125}$ (B) $\frac{3}{5}$ (C) $\frac{16}{25}$ (D) $\frac{1}{5}$ (E) $\frac{12}{125}$

This is an example seen in class. With n keys the probability is 1/n. Here n=5.

1. A fair die is tossed until either a 5 or 6 appears. What is the probability that we will need to roll the die exactly 3 times before observing a 5 or 6? On lance un de equilibre jusqu'a ce qu'on obtiene un 5 ou un 6. Quelle est la probabilite qu'il faudra lancer le de 3 fois?.
   (A) $\frac{2}{27}$ (B) $\frac{4}{27}$ (C) $\frac{8}{27}$ (D) $\frac{12}{27}$ (E) $\frac{1}{3}$

The probability of a 5 or a 6 with a fair die is p = 1/6 +1/6 =1/3

Now if X denotes the number of tosses required, the density of X is geometric with parameter p = 1/3

Hence P (X=3) = (1-p)^2p = $(2/3)^2$ (1/3) = 4/27

1. The percentage additive to a certain type of gasoline is a random variable with density given.

$$\begin{aligned} f_X\left(x\right) &= kx^3\left(1-x\right), 0<x<1 \\ &= 0, otherwise. \end{aligned}$$

   Calculate the constant k.

   (A) 20 (B) $\frac{1}{20}$ (C) $\frac{256}{27}$ (D) $\frac{27}{256}$ (E) 1

Since we know we have a density, it must integrate to 1. Hence,

$$\int_0^1 kx^3 (1-x)\, dx = k/20 = 1$$

Consequently k = 20

1. A professor receives on average 5 emails per hour. Given that the number of emails he receives follows a Poisson distribution, what is the probability that he will receive at most 20 emails in 3 hours? Un professeur recoit en moyenne 5 courriels par heure. Si le nombre de courriels recu suit une distribution Poisson, quelle est la probabilite qu'il recoive au plus 20 courriels dans 3 heures?

   (A) $\frac{5}{20}$ (B) 1 (C) 0.083 (D) 0.917 (E) $\frac{3}{5}$

This is a direct application of the Poisson distribution. He receives $\mu = 15$ emails on average in 3 hours. From the table of the Poisson $F(20) = 0.917$

1. A random variable X has the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{20} & 0 \le x < 1 \\ \frac{5}{20} & 1 \le x < 2 \\ \frac{10}{20} & 2 \le x < 3 \\ \frac{15}{20} & 3 \le x < 4 \\ 1 & 4 \le x \end{cases}$$

   Calculate the conditional probability $P(X \geq 3|X > 1)$.

   (A) $\frac{1}{20}$ (B) $\frac{5}{20}$ (C) $\frac{10}{20}$ (D) $\frac{2}{3}$ (E) $\frac{15}{20}$

$$\begin{aligned} P(X \geq 3|X > 1) &= \frac{P(X \geq 3, X > 1)}{P(X > 1)} \\ &= \frac{P(X \geq 3)}{1 - P(X \leq 1)} \\ &= \frac{1 - P(X < 3)}{1 - 5/20} \\ &= \frac{1 - 10/20}{15/20} = 2/3 \end{aligned}$$

1. We know that for independent random variables X, Y, $E(X) = 2, E(Y) = 4$,$\sigma_X^2 = 1, \sigma_Y^2 = 5$.

   Calculate i) $E(2X + 3Y)$,$ii)Variance(2X + 3Y)$.
   (A) i)16; ii) 49 (B) i)16; ii)17 (C)i)6; ii) 49 (D) i)16; ii) $2{+}3\sqrt{5}$ (E) i)16; ii) 16

i) $E(2X + 3Y) = 2E(X) + 3E(Y) = 2(2) + 3(4) = 16$
ii) $Var(2X + 3Y) = 2^2 Var(X) + 3^2 Var(Y) = 4(1) + 9(5) = 49$

1. The density of a discrete random variable X is given by

x	0	1	2	3	4
$f(x)$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{2}{10}$

   Calculate $P(1.5 \leq X < 3.9)$

   (A) $\frac{1}{10}$ (B) $\frac{2}{10}$ (C) $\frac{5}{10}$ (D) 0 (E) $\frac{4}{20}$

Using the density we note that $P(1.5 \leq X < 3.9) = P(X = 2, 3) = 1/10 + 2/10 = 5/10$

1. Calculate the mean $E(X)$ in the question above.

   (A) 20 (B) 2 (C) 1 (D) 10 (E) 0

$\mu = \sum xf(x) = 0(2/10) + 1(1/10) + 2(4/10) + 3(1/10) + 4(2/10) = 2$

1. Let X, Y be the number of hand produced bicycles by two workers A, B respectively in a single day. The joint density is given below. Compute $P(X \geq 1)$.

x\y	0	1	2	3
0	0.00	0.05	0.10	0.10
1	0.05	0.10	0.10	0.10
2	0.10	0.10	0.10	0.10

   (A) 0.35 (B) 0.40 (C) 0.75 (D) 0.25 (E) 0.30

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - (P(X = 0, Y = 0) + P(X = 0, Y = 1) + P(X = 0, Y = 2) + P(X = 0, Y = 3 \\ &= 1 - 0.25 = 0.75 \end{aligned}$$

1. In the question above, calculate the probability that A produces more bicycles than B; that is $P(X > Y)$

   (A) 0.05 (B) 0.10 (C) 0.15 (D) 0.20 (E) 0.25

$P(X > Y) = P(X = 1, Y = 0) + P(X = 2, Y = 0) + P(X = 2, Y = 1) = 0.05 + 0.10 + 0.10 = 0.25$

1. An electronic system contains 4 components in parallel. The entire system functions if at least one of the components functions. The probability that component i does not function is denoted by p_i, $i = 1, 2, 3, 4$ where $p_1 = .1, p_2 = .2, p_3 = .1, p_4 = .25$. If all the components function independently of one another, what is the probability that the entire system functions?

   Un sytème électronique contient 4 composantes en parallèle. Le système fonctionne quand au moins une composante fonctionnne. La probabilité que la i^e composante ne fonctionne pas est denotée par p_i, $i = 1, 2, 3, 4$, où $p_1 = .1, p_2 = .2, p_3 = .1, p_4 = .25$. Si les composantes fonctionnent indépendamment l'une de l'autre, quelle est la probabilité que le système fonctionne?

   A) 0.0005 (B) 0.9995 (C) 0.4860 (D) 0.6500 (E) 0.3500

P(all function) = 1-P(none function) =1-0.01(0.2) (0.1) (0.25) = 0.9995