Abstracts

(as of May 4, 2019)

Mini-courses:

Martina Lanini
Moment graphs and Schubert calculus

In this mini-course we will start by discussing some of the enumerative geometric questions from which Schubert calculus was born. We will then introduce Schubert varieties, and rephrase the above questions in terms of these varieties. Finally, we will see how to construct a graph from a torus action on a (nice) variety, apply this contruction to the Grassmannian, and attack our original questions in these terms.

Alexander Merkurjev
Introduction to generic forms and flags

Let G be a linear algebraic group over a field F.
We will be discussing the following two structures:
1. The representation ring R(G) and the three filtrations on it.
2. The classifying space BG and the rationality problem."

50 min. talks:

David Anderson
Old formulas for degeneracy loci, with a new twist

A basic problem from the 19th century asks for the degree of the locus of symmetric matrices of bounded rank; answers were given by Schubert and Giambelli. More recently, many extensions of this problem have been considered, including versions coming from symmetric maps of vector bundles, or for vector bundles equipped with a nondegenerate bilinear form. I will discuss ongoing work with William Fulton, in which we allow the bilinear form to be “twisted”, so that it takes values in a nontrivial line bundle. The formulas we obtain extend those of Billey-Haiman, Ikeda-Mihalcea-Naruse, and others, and exhibit new connections with algebraic combinatorics.

Ben Elias
Frobenius structures and the exotic nilHecke algebra

The nilHecke algebra is a family of operators on the polynomial ring in n variables, which are each linear over symmetric polynomials. It arises naturally in Schubert calculus and the study of the cohomology rings of flag varieties. In fact, various

properties of the polynomial ring and the nilHecke algebra are actually shadows of deeper geometric theorems. For example, the Chevalley theorem states that the polynomial ring is free as a module over symmetric polynomials, but Poincare duality can be adapted to prove an upgraded version: that the polynomial ring is a Frobenius extension over symmetric polynomials. NilHecke algebras are key tools towards studying this Frobenius structure, because they allow one to describe the Frobenius trace explicitly.
There are numerous situations which mimic geometry and seem to have all the same wonderful structures, without having any actual underlying geometry. For example, Chevalley's theorem and its upgrade hold for any finite Coxeter group. In this talk we'll discuss and explore a new quasi-geometric setting which is still work in progress: the q-deformed exotic nilHecke algebra, which is related to quantum geometric Satake. If you like Schubert calculus and want something harder, you'll enjoy this!

Peter Fiebig
Arithmetic Properties of affine moment graphs

Affine moment graphs exhibit quite remarkable arithmetic properties. In the talk I want to show some examples, and connect the emerging structure to the problem of determining Weyl module multiplicities for tilting modules of reductive algebraic groups.

Jacopo Gandini
Nilpotent B-orbits associated to sets of orthogonal roots

Let G be a semisimple algebraic group, with a fixed Borel subgroup B and maximal torus T in B. To any set of pairwise strongly orthogonal roots I will attach a nilpotent B-orbit in the Lie algebra of G, and will explain how the combinatorics of the involutions in the Weyl group of G is related to the geometry of these B-orbits. The talk is based on joint works with A. Maffei, P. Moseneder Frajria and P. Papi.

Jochen Kuttler
Peterson translates and Singularities of Schubert varieties

In this talk I will survey the theory of Peterson Translates as it applies to singularities of Schubert varieties in both the classical and affine setting.
This is joint work with Valerie Budd, Jim Carrell, and Andrew Crites

Cristian Lenart
Generalizing Schubert calculus to hyperbolic cohomology and quantum K-theory

Schubert calculus has been mostly concerned with the study of the (equivariant) cohomology (or quantum cohomology) and K-theory of flag varieties. This talk has two parts, focusing on the two generalizations in the title.
Hyperbolic cohomology is the simplest example of a generalized cohomology theory beyond K-theory. Thus, in the case of a flag variety, there is no canonical geometric basis (or Schubert basis). However, a canonical basis was constructed algebraically in joint work with K. Zainoulline and C. Zhong. We provide geometric motivation for our construction.
Quantum K-theory is a K-theoretic version of quantum cohomology. I present a related Chevalley-type formula for expanding in the Schubert basis the product of an arbitrary Schubert class and the class of a line bundle. This is joint work with S. Naito and D. Sagaki, whose starting point is a recent geometric result of S. Kato.

Elizabeth Milicevic
The Peterson Isomorphism: Moduli of Curves and Alcove Walks

In this talk, I will explain the combinatorial tool of folded alcove walks, in addition to surveying a wide range of applications in combinatorics, representation theory, and algebraic geometry. As a concrete example, I will describe a labeling of the points of the moduli space of genus 0 curves in the complete complex flag variety using the combinatorial machinery of alcove walks. Following Peterson, this geometric labeling partially explains the "quantum equals affine" phenomenon which relates the quantum cohomology of this flag variety to the homology of the affine Grassmannian. This is joint work with Arun Ram.

Nicole Lemire
Schubert Cycles and Subvarieties of Twisted Projective Homogeneous Varieties

Twisted projective homogeneous varieties for algebraic groups are projective varieties which are isomorphic to a given projective homogeneous variety after extension to the separable closure of the field. Important examples include Severi-Brauer varieties, which are twisted forms of projective space; generalised Severi-Brauer varieties, which are twisted forms of Grassmannians. We investigate conditions under which generalised Severi-Brauer varieties have rational subvarieties which are forms of given Schubert subvarieties of the associated Grassmannian. For regular Grassmannians, and more generally for any projective homogeneous variety for an algebraic group, the classes of the Schubert subvarieties of a given codimension give a basis for the Chow groups of that codimension. The Chow groups, even in codimension 2, for twisted projective homogeneous varieties, are less well understood. We use our results as well as an analysis of the Chow motive and the topological filtration of the Grothendieck group of generalised Severi-Brauer varieties to show that the

codimension 2 Chow groups of certain generalised Severi-Brauer varieties are torsion free.
This is joint work with Caroline Junkins and Danny Krashen.

In joint work with Caroline Junkins and Jasmin Omanovic, we investigate the analogous problem for Lagrangian involution varieties, twisted forms of Lagrangian Grassmannians. We determine conditions under which Lagrangian involution varieties have rational subvarieties which are forms of given Schubert subvarieties of the associated Lagrangian Grassmannian. We also investigate the torsion in the codimension 2 Chow groups of certain Lagrangian involution varieties.

Victor Petrov
Motives of projective homogeneous varieties: Hopf-theoretic approach

Let G be a split semisimple linear algebraic group over a field F and E be a G-torsor over F. We show that the realization functor from the category of motives of E-twisted forms of projective G-varieties preserves the structure of coaction of some bialgebra naturally associated to E (and depending only on J-invariant of E). We demonstrate how this additional structure can be used to obtain some new results and simpler proofs of known results on motivic decompositions of projective homogeneous varieties."

30 min. talks:

Seidon Alsaody
Compositions of quadratic forms and exceptional Jordan algebras

Exceptional Jordan algebras, known as Albert algebras, have dimension 27 and automorphism groups of type F4. Over fields, they are either division algebras or are reduced, i.e. arise as (twisted) hermitian matrices over octonion algebras.
A classical result by Albert and Jacobson asserts that a reduced Albert algebra over a field determines the involved octonion algebra completely. I will talk about what the situation looks like over general commutative, unital rings. This will involve compositions of quadratic forms, triality and certain related torsors.

Lara Bossinger
Full rank valuations and toric initial ideals

Let V(I) be a polarized projective variety or a subvariety of a product of projective spaces and let A be its (multi-)homogeneous coordinate ring. Given a full-rank valuation v on A we associate weights to the coordinates of the projective space, respectively, the product of projective spaces. Let wv be the vector whose entries are these weights. Our main result is that the value semi-group of v is generated by the images of the generators of A if and only if the initial ideal of I with respect to wv is prime. We further show that wv always lies in the tropicalization of I. Applying our result to string valuations for flag varieties, we solve a conjecture connecting the Minkowski property of string cones with the tropical flag variety. For Rietsch-Williams' valuation for Grassmannians our results give a criterion for when the Plücker coordinates form a Khovanskii basis. Further, as a corollary we obtain that the BFFHL-weight vectors lie in the tropical Grassmannian.

Nicolas Garrel
Exterior powers of hermitian forms over algebras with involution

We introduce an appropriate notion of exterior powers for hermitian forms over central simple algebras with involution of the first kind, which generalize exterior powers of bilinear forms, using certain canonical Morita equivalences. They define a pre-lambda-ring structure on some "mixed" Witt ring attached to the algebra with involution. We wish to discuss in particular the relations that these exterior powers may satisfy, and the potential relations with the representation theory of non-split classical groups.

Eoin Mackall
Cycles of large codimension on some twisted flag varieties

Computing the Chow groups of a twisted flag variety has been both a highly motivating, and highly difficult, problem for a number of decades.
In recent years, this problem has begun yielding in some cases of generic twisted flag varieties. For example, recent results of Karpenko show that the Chow ring of a generic Severi-Brauer variety is isomorphic to a graded ring associated to the computable Grothendieck ring of the same variety.
In this talk, we show how one can use this result to compute the Chow groups of cycles of large codimension on any generic twisted flag variety of type A which has the additional property of being generically split.

Leonardo Patimo
Dyck partitions and the intersection cohomology of Schubert varieties

The cohomology of Grassmannians is a very classical object of study and we now understand it well in terms of its distinguished basis: the Schubert basis.
Understanding the intersection cohomology of Schubert variety in a Grassmannian is a much more difficult task. The related Kazhdan-Lusztig polynomials, however, have been widely studied from a combinatorial perspective: they can be computed, for example, by counting certain Dyck partitions.
In this talk we will explain how, by ""lifting"" the rich combinatorics of this Dyck partitions, we are able to extend the Schubert basis to a basis of the intersection cohomology.